ELEC0047 - Power system dynamics, control and stability

Small-disturbance angle stability analysis and improvement

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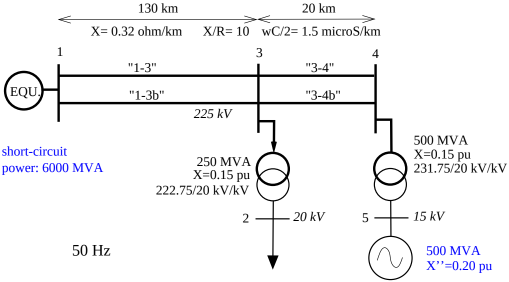
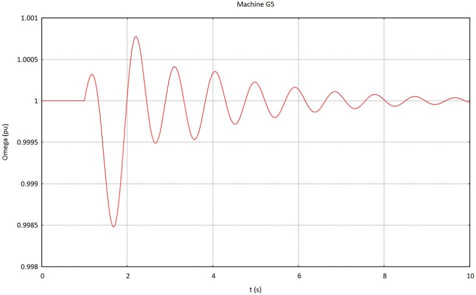
Examples of electromechanical oscillations

The 5-bus system (studied in previous lectures) illustrating a *local plant mode*



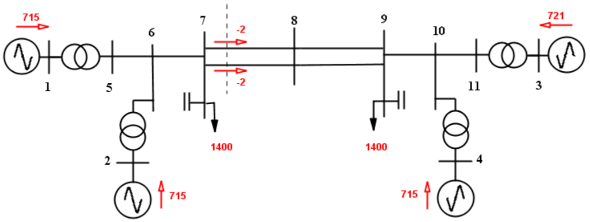
oscillation of one machine against the rest of the system



response of machine rotor speed to a 10 % drop of Thévenin e.m.f. lasting 0.05 s

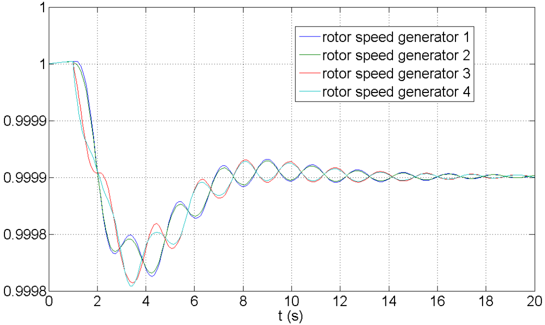
period $\simeq$ 1 s

The “Kundur” test system illustrating an *interarea oscillation*



oscillation of machines 1 and 2 against machines 3 and 4

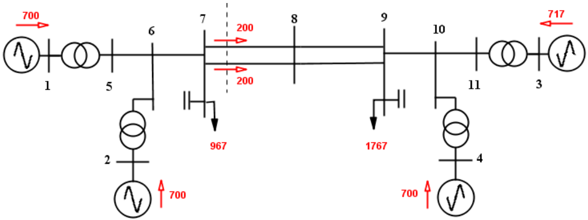
(almost) no power flow between left and right parts



response of machine rotor speeds
to a 1 % increase of load at bus 9

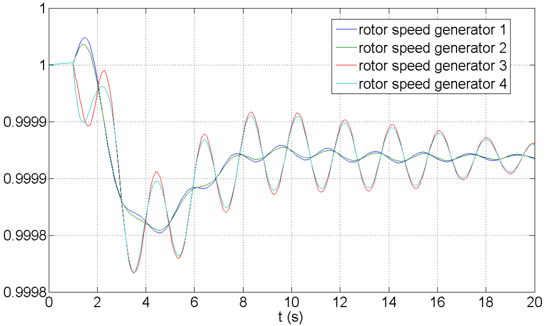
period $\simeq$ 2 s

The "Kundur" test system illustrating an *interarea oscillation*



400 MW power flow from left and to right part

oscillation of machines 1 and 2 against machines 3 and 4 with a different *mode shape*



response of machine rotor speeds
to a 1 % increase of load at bus 9

period $\simeq$ 2 s

Classification of oscillation modes

Local modes: involve a small part of the system

- rotor angle oscillations of a single generator or a single plant against the rest of the system: *local plant mode*
    - can be studied using a one-machine infinite-bus system
- oscillations between rotors of a few generators close to each other: *intermachine or interplant mode oscillations*
- typical range of frequencies of local plant and interplant modes: 0.7 to 2 Hz
- may also be associated with inappropriate tuning of a control equipment (excitation system, HVDC converter, SVC, etc.): *control mode*

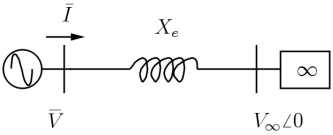
Global modes: involve large areas of the system, widespread effects

- oscillations of a large goup of generators in one area swinging against a group of generators in another area: *interarea mode*
- usually, the larger the group of generators, the slower the oscillations
- typical range of frequencies of interarea modes: 0.1 to 0.7 Hz
- more complex to analyse and to damp

Objectives of this lecture

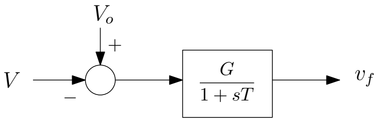
- Present an approach to analyse the small-disturbance stability of a system described by an algebraic-differential model
- apply this approach to a simple one-machine infinite-bus system (typical of a local plant mode study)
  - derive the model
  - compute (one of) its equilibrium point(s)
  - analyze the small-disturbance stability of this equilibrium
- present a method to improve small-disturbance stability by correcting troublesome eigenvalues
- apply this method to the design of a "power system stabilizer" acting on the one-machine infinite-bus system
- practice this with Matlab.

Model of a one-machine infinite-bus system

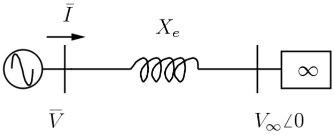


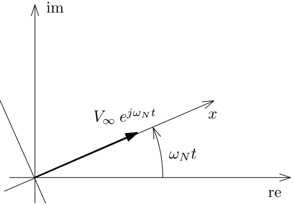
Simplifying assumptions

- d axis: only the field winding is considered
- q axis: only one winding (q_1) is considered, to simulate a damper
- the stator resistance is neglected
- saturation is neglected
- mechanical torque T_m is considered constant
- rotor speed remains close to nominal value: $\omega \simeq 1$ pu
- a very simple Automatic Voltage Regulator (AVR) model is considered:



Network equations under the phasor approximation





Phasor of voltage at infinite bus: zero phase angle.

(x, y) reference axes: rotate at nominal angular speed ω_N, and the axis x coincides with the rotating vector relative to the infinite bus voltage.

$$\bar{V} = V_\infty \angle 0 + jX_e \bar{I} \quad \Leftrightarrow \quad v_x + jv_y = V_\infty + jX_e \left(i_x + j\, i_y \right)$$

Decomposing in real and imaginary parts:

$$v_x = V_\infty - X_e i_y \qquad (1)$$
$$v_y = X_e i_x \qquad (2)$$

All variables and parameters are in per unit.

Park equations of synchronous machine under the phasor approximation

$$\psi_d = L_{dd}\, i_d + L_{df}\, i_f \quad (3)$$

$$\psi_q = L_{qq}\, i_q + L_{qq1}\, i_{q1} \quad (4)$$

$$\psi_f = L_{ff}\, i_f + L_{df}\, i_d \quad (5)$$

$$\psi_{q1} = L_{q1q1}\, i_{q1} + L_{qq1}\, i_q \quad (6)$$

$$\frac{1}{\omega_N}\,\frac{d}{dt}\psi_f = K\, v_f - R_f\, i_f \quad (7)$$

$$\frac{1}{\omega_N}\,\frac{d}{dt}\psi_{q1} = -R_{q1}\, i_{q1} \quad (8)$$

$$v_d = -\omega\psi_q = -\psi_q \quad (9)$$

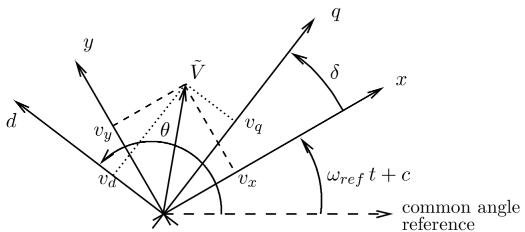
$$v_q = \omega\psi_d = \psi_d \quad (10)$$

$$2H\,\frac{d}{dt}\omega = T_m - T_e = T_m - (\psi_d\, i_q - \psi_q\, i_d) \quad (11)$$

$$\frac{1}{\omega_N}\,\frac{d}{dt}\delta = \omega - 1 \quad (12)$$

- All variables are in per unit, except δ which is in rad and t in seconds. Hence, the factor $t_B = 1/\omega_N$ in Eqs. (7, 8, 12), where ω_N is in rad/s.
- All parameters are in per unit, except H which is in seconds.
- v_f is in per unit on the AVR voltage base. K is a factor to pass from the AVR base to the machine base, which is used in Eq. (7).

Change of reference axes: from machine (d, q) to system (x, y) reference



in this system: $c = 0$ and $\omega_{ref} = \omega_N$

$$v_q + jv_d = e^{-j\delta}(v_x + j\, v_y) = (\cos\delta - j\, \sin\delta)(v_x + j\, v_y)$$

Decomposing into real and imaginary components:

$$v_d = -\sin\delta\, v_x + \cos\delta\, v_y \quad (13)$$

$$v_q = \cos\delta\, v_x + \sin\delta\, v_y \quad (14)$$

Similarly for the current:

$$i_d = -\sin\delta\, i_x + \cos\delta\, i_y \quad (15)$$

$$i_q = \cos\delta\, i_x + \sin\delta\, i_y \quad (16)$$

Automatic voltage regulator

$$\frac{d}{dt} v_f = \frac{-v_f + G(V_o - V)}{T} = \frac{-v_f + G(V_o - \sqrt{v_x^2 + v_y^2})}{T} \qquad (17)$$

- t and T are in seconds
- V_o, V, v_x and v_y are in per unit on the network base voltage
- v_f is in per unit on the AVR voltage base V_{fB}
- commonly used base: V_{fB} = field voltage that produces $V = 1$ pu at the terminal of the machine rotating at nominal speed with stator open.

$$i_d = i_q = 0 \quad \Rightarrow \quad \psi_d = L_{df} i_f \text{ and } \psi_q = 0 \quad \Rightarrow \quad v_q = L_{df} i_f \text{ and } v_d = 0$$

$$\Rightarrow \quad V = 1 = \sqrt{v_d^2 + v_q^2} = L_{df} i_f \quad \Rightarrow \quad i_f = \frac{1}{L_{df}}$$

$$\frac{d\psi_f}{dt} = 0 \text{ and } v_f = 1 \quad \Rightarrow \quad K = R_f i_f = \frac{R_f}{L_{df}}$$

- G is the AVR open-loop gain in pu/pu

Variables and equations are balanced

17 variables:

- 5 differential: $\psi_f, \psi_{q1}, \omega, \delta, v_f$
- 12 algebraic: $v_x, v_y, i_x, i_y, v_d, v_q, i_d, i_q, i_f, i_{q1}, \psi_d, \psi_q$

17 equations:

- 5 differential: Eqs. (7, 8, 11, 12, 17)
- 12 algebraic: Eqs. (1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 15, 16)

Comments

- In this simple system, some or all algebraic variables (and an equal number of algebraic equations) could be eliminated, thus yielding a smaller model;
- however, the techniques shown hereafter do not require performing such manipulations;
- on the contrary, keeping them in the model allow us to illustrate how differential-algebraic models are treated in practice.

Small-disturbance stability analysis

Linearization of a system described by differential equations

Consider a system described by the differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \qquad\qquad \dim\ \mathbf{x} = \dim\ \mathbf{f} = n$$

Let $\mathbf{x}^\star$ be an equilibrium point : $\qquad \mathbf{f}(\mathbf{x}^\star) = \mathbf{0}$

Consider an infinitesimal variation of $\mathbf{x}$ around $\mathbf{x}^\star$: $\qquad \Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^\star$

The dynamics of $\Delta\mathbf{x}$ is given by:

$$\dot{\Delta\mathbf{x}} = \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \simeq \mathbf{f}(\mathbf{x}^\star) + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big)_{\mathbf{x}^\star}(\mathbf{x} - \mathbf{x}^\star) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big)_{\mathbf{x}^\star}\Delta\mathbf{x}$$

where $\dfrac{\partial \mathbf{f}}{\partial \mathbf{x}}$ is the *Jacobian of* $\mathbf{f}$ *with respect to* $\mathbf{x}$: $\quad \left[\dfrac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{ij} = \dfrac{\partial f_i}{\partial x_j} \quad i, j = 1, \ldots, n$

$\dfrac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big)_{\mathbf{x}^\star}$ is the *state matrix* of the linearized system.

Stability of the equilibrium $\mathbf{x}^\star$

Assessed from the n eigenvalues of $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big)_{\mathbf{x}^\star}$:

- if all eigenvalues have negative real parts, $\mathbf{x}^\star$ is stable;
  $\mathbf{x}^\star$ is a *sink* or *stable node*
- if at least one eigenvalue has a positive real part, $\mathbf{x}^\star$ is unstable
  - all eigenvalues with positive real part: $\mathbf{x}^\star$ is a *source* or *unstable node*
  - some eigenvalues with positive real part: $\mathbf{x}^\star$ is a *saddle*.
- if the eigenvalues have negative real parts, except some of them which have a zero real part, stability cannot be decided; higher-order terms of the Taylor series expansion have to be investigated.

In practice, to have some "margin" with respect to instability, the eigenvalues must be "at some distance" from the right half complex plane.

Linearization of a system described by differential-algebraic equations

Consider a system described by the algebraic-differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}) \qquad \dim \mathbf{x} = \dim \mathbf{f} = n \qquad (18)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}) \qquad \dim \mathbf{g} = \dim \mathbf{y} = m \qquad (19)$$

Let $(\mathbf{x}^\star, \mathbf{y}^\star)$ be an equilibrium point :

$$\mathbf{0} = \mathbf{f}(\mathbf{x}^\star, \mathbf{y}^\star)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}^\star, \mathbf{y}^\star)$$

Implicit function theorem. Let $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ be the Jacobian of $\mathbf{g}$ with respect to $\mathbf{y}$. At a point $(\mathbf{x}, \mathbf{y})$ where $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular, there exists a unique and differentiable function φ such that locally:

$$\mathbf{y} = \varphi(\mathbf{x})$$

Then, substituting $\varphi(\mathbf{x})$ to $\mathbf{y}$ in (18) yields the differential (only) model:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \varphi(\mathbf{x})) = \mathbf{F}(\mathbf{x})$$

Stability has to be studied on the $(n \times n)$ Jacobian of $\mathbf{F}$ with respect to $\mathbf{x}$!

- Except in simple cases, the analytical expression of φ cannot be derived
- instead, the differential-algebraic model is linearized and the algebraic states are eliminated, as shown next.

With the same notation as in the previous slides:

$$\dot{\Delta \mathbf{x}} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)_{\mathbf{x}=\mathbf{x}^\star,\mathbf{y}=\mathbf{y}^\star} \Delta \mathbf{x} + \left.\frac{\partial \mathbf{f}}{\partial \mathbf{y}}\right)_{\mathbf{x}=\mathbf{x}^\star,\mathbf{y}=\mathbf{y}^\star} \Delta \mathbf{y} \quad (20)$$

$$\mathbf{0} = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right)_{\mathbf{x}=\mathbf{x}^\star,\mathbf{y}=\mathbf{y}^\star} \Delta \mathbf{x} + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)_{\mathbf{x}=\mathbf{x}^\star,\mathbf{y}=\mathbf{y}^\star} \Delta \mathbf{y} \quad (21)$$

Assuming that $\left.\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)_{\mathbf{x}=\mathbf{x}^\star,\mathbf{y}=\mathbf{y}^\star}$ is nonsingular[1]:

$$\Delta \mathbf{y} = -\left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1} \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \Delta \mathbf{x} \quad \Rightarrow \quad \dot{\Delta \mathbf{x}} = \underbrace{\left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1} \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]}_{\mathbf{A}} \Delta \mathbf{x}$$

Stability is analyzed from the eigenvalues of the *reduced Jacobian* **A**.

[1] the dependency on the linearization point is omitted for simplicity of notation

What if $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is singular ?

A point $(\mathbf{x}, \mathbf{y})$ where $\frac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is singular is called a *singularity*.

- For a small variation of $\mathbf{x}$, the variation of $\mathbf{y}$ becomes infinitely large
- the system dynamics becomes undefined; numerical integration cannot proceed
- the mathematical model stops matching the physical system (whose time evolution cannot stop !)
- singularities often originate from model simplifications: some dynamics assumed infinitely fast and replaced by algebraic equilibrium conditions.

Extension to model with inputs and outputs

Consider a system described by:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \quad (22)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \quad (23)$$

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \quad (24)$$

where $\mathbf{u}$ is a vector of *inputs (or controls)* and $\mathbf{z}$ of *outputs (or measurements)*.

After linearization:

$$\dot{\Delta \mathbf{x}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{y}}\Delta \mathbf{y} + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\Delta \mathbf{u} \quad (25)$$

$$\mathbf{0} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\Delta \mathbf{x} + \frac{\partial \mathbf{g}}{\partial \mathbf{y}}\Delta \mathbf{y} + \frac{\partial \mathbf{g}}{\partial \mathbf{u}}\Delta \mathbf{u} \quad (26)$$

$$\Delta \mathbf{z} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}\Delta \mathbf{x} + \frac{\partial \mathbf{h}}{\partial \mathbf{y}}\Delta \mathbf{y} + \frac{\partial \mathbf{h}}{\partial \mathbf{u}}\Delta \mathbf{u} \quad (27)$$

Extracting $\Delta \mathbf{y}$ from (26) and substituting in (25,27):

$$\dot{\Delta \mathbf{x}} = \underbrace{\left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}}\left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1}\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]}_{\mathbf{A}}\Delta \mathbf{x} + \underbrace{\left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}}\left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1}\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right]}_{\mathbf{B}}\Delta \mathbf{u} \quad (28)$$

$$\Delta \mathbf{z} = \underbrace{\left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} - \frac{\partial \mathbf{h}}{\partial \mathbf{y}}\left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1}\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]}_{\mathbf{C}}\Delta \mathbf{x} + \underbrace{\left[\frac{\partial \mathbf{h}}{\partial \mathbf{u}} - \frac{\partial \mathbf{h}}{\partial \mathbf{y}}\left(\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right)^{-1}\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right]}_{\mathbf{D}}\Delta \mathbf{u} \quad (29)$$

Standard form of a linear system with inputs and outputs.

Exercises using MATLAB

System Data

All values in per unit refer to the nominal apparent power of the machine

$$X_e = 0.45 \; pu \qquad f_N = 50 \; Hz$$

Synchronous machine:

$$X_\ell = 0.2 \text{ [2]} \qquad X_d = 2.2 \qquad X_d' = 0.3 \qquad X_q = 2.2 \; pu \qquad X_q' = 0.25 \; pu$$

$$T_{do}' = 7.0 \; s \qquad T_{qo}' = 0.4 \; s \qquad H = 4 \; s$$

Automatic Voltage Regulator:

$$T = 0.4 \; s \qquad G = 70 \; pu/pu$$

Operating point: voltage, active and reactive powers of machine specified

$$V = 1 \; pu \qquad P = 0.7 \; pu \qquad Q = 0.15 \; pu$$

[2] X_ℓ is the leakage reactance

Deriving the inductances and resistances of the model (EMFL pu system)

$$L_{dd} = X_d = 2.2 \qquad L_{qq} = X_q = 2.2 \; pu$$

$$t_B = \frac{1}{\omega_N} = \frac{1}{2\pi f_N} = 3.183\,10^{-3} \; s$$

$$T'_{do\;pu} = \frac{T'_{do\;s}}{t_B} = 2199 \; pu \qquad T'_{qo\;pu} = \frac{T'_{qo\;s}}{t_B} = 125.7 \; pu$$

$$L_{df} = L_{dd} - X_\ell = 2.0 \qquad L_{qq1} = L_{qq} - X_\ell = 2.0 \; pu$$

$$X'_d = L'_d = L_{dd} - \frac{L_{df}^2}{L_{ff}} \quad \Rightarrow \quad L_{ff} = \frac{L_{df}^2}{L_{dd} - X'_d} = 2.105 \; pu$$

$$X'_q = L'_q = L_{qq} - \frac{L_{qq1}^2}{L_{q1q1}} \quad \Rightarrow \quad L_{q1q1} = \frac{L_{qq1}^2}{L_{qq} - X'_q} = 2.051 \; pu$$

$$T'_{do} = \frac{L_{ff}}{R_f} \quad \Rightarrow \quad R_f = \frac{L_{ff}}{T'_{do}} = 9.573\,10^{-4} \; pu$$

$$T'_{qo} = \frac{L_{q1q1}}{R_{q1}} \quad \Rightarrow \quad R_{q1} = \frac{L_{q1q1}}{T'_{qo}} = 1.632\,10^{-4} \; pu$$

$$K = \frac{R_f}{L_{df}} = 4.787\,10^{-4} \; pu$$

Values of the differential and algebraic states at the operating point

$$V_\infty = 0.9843 \qquad v_x = 0.9474 \qquad v_y = 0.3200\ pu$$

$$i_x = 0.7112 \qquad i_y = 0.0819\ pu \qquad \delta = 1.1842\ rad$$

$$i_d = -0.6278 \qquad i_q = 0.3440 \qquad v_d = -0.7568 \qquad v_q = 0.6536\ pu$$

$$\psi_d = 0.6536 \qquad \psi_q = 0.7568 \qquad \psi_f = 0.8863 \qquad \psi_{q1} = 0.6880\ pu$$

$$i_f = 1.0174 \qquad i_{q1} = 0 \qquad v_f = 2.0348\ pu$$

<u>Short exercises</u>

- Check that these values of v_x, v_y, i_x and i_y yield $P = 0.7$ and $Q = 0.15$ pu
- same question using v_d, v_q, i_d and i_q
- compute the electromagnetic torque T_e in pu. Comment on its value
- compute the magnitude of the e.m.f E_q behind synchronous reactances (i) from the value of i_f; (ii) from the equation: $\bar{E}_q = \bar{V} + jX_d\bar{I}_d + jX_q\bar{I}_q$

The Matlab script `omib.m`

- derives the inductances and resistances of the model as shown in slide # 20
- computes the state variables at the operating point as shown in slide # 21
- computes the "full" Jacobian:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{\partial \mathbf{f}}{\partial \mathbf{y}} & \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}}{\partial \mathbf{y}} & \frac{\partial \mathbf{g}}{\partial \mathbf{u}} \\ \frac{\partial \mathbf{h}}{\partial \mathbf{x}} & \frac{\partial \mathbf{h}}{\partial \mathbf{y}} & \frac{\partial \mathbf{h}}{\partial \mathbf{u}} \end{bmatrix}$$

  for the particular case:

$$z = \omega \qquad u = V_o$$

- computes the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ of the linearized system

Exercise # 1

- Compute the system eigenvalues assuming: constant flux ψ_f, constant flux ψ_{q1} and constant voltage v_f
- compute the system eigenvalues assuming: constant flux ψ_{q1} and constant voltage v_f. Comment on the influence of the field winding
- compute the system eigenvalues assuming constant voltage v_f. Comment on the influence of the $q1$ (damper) winding
- compute the system eigenvalues under AVR control
- compare the period of electromechanical oscillations in all four cases.

Exercise # 2

- How do the system eigenvalues evolve when increasing the active power to $P = 0.9$ pu (leaving Q and V unchanged) ?
- How do the system eigenvalues evolve when increasing the gain G from 70 to 120 (keeping $T = 0.4$ s) ?
- How do the system eigenvalues evolve when decreasing the time constant T from 0.4 to 0.1 s (keeping $G = 70$) ?

Exercise # 3

With $P = 0.9$ pu and $V = 1$ pu, determine, with an accuracy of 0.05 pu, the maximum reactive power that the generator :

- can produce
- can absorb

without the operating point becoming unstable, under AVR control.

Which eigenvalues become unstable ?

Design of a stabilizing feedback by the method of residues

Left and right eigenvectors

Consider an $n \times n$ matrix $\boldsymbol{A}$ with all distinct and nonzero eigenvalues λ_i.

Let $\boldsymbol{v}_i$ be the *right eigenvector*[3] of λ_i :

$$\boldsymbol{A}\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i \quad (i = 1, \dots, n)$$

and $\boldsymbol{w}_i$ the *left eigenvector* of λ_i :

$$\boldsymbol{w}_i^T \boldsymbol{A} = \lambda_i \boldsymbol{w}_i^T \quad \Leftrightarrow \quad \boldsymbol{A}^T \boldsymbol{w}_i = \lambda_i \boldsymbol{w}_i \quad (i = 1, \dots, n)$$

Let $\boldsymbol{V}$ (resp. $\boldsymbol{W}$) be the matrix of right (resp. left) eigenvectors:

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{v}_1 & \dots & \boldsymbol{v}_n \end{bmatrix} \qquad \boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1^T \\ \vdots \\ \boldsymbol{w}_n^T \end{bmatrix}$$

It is easily shown that : $\boldsymbol{W} = \boldsymbol{V}^{-1}$ and $\boldsymbol{WAV} = diag(\lambda_i)$

[3] notation: all vectors are column vectors

Controllability and observability of a mode

Consider a system with state vector $\boldsymbol{x}$, a scalar input u and a scalar output y :

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \qquad z = \boldsymbol{c}^T\boldsymbol{x} + du$$

Consider the change of variables : $\tilde{\boldsymbol{x}} = \boldsymbol{W}\boldsymbol{x}$.

$$\boldsymbol{W}^{-1}\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\boldsymbol{W}^{-1}\tilde{\boldsymbol{x}} + \boldsymbol{b}u \qquad z = \boldsymbol{c}^T\boldsymbol{W}^{-1}\tilde{\boldsymbol{x}} + du$$

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{W}^{-1}\tilde{\boldsymbol{x}} + \boldsymbol{W}\boldsymbol{b}u = \boldsymbol{\Lambda}\tilde{\boldsymbol{x}} + \boldsymbol{W}\boldsymbol{b}u \qquad z = \boldsymbol{c}^T\boldsymbol{V}\tilde{\boldsymbol{x}} + du$$

$$\text{where } \boldsymbol{\Lambda} = diag(\lambda_i)$$

For the i-th "mode" λ_i :

- the larger $(\boldsymbol{W}\boldsymbol{b})_i = \boldsymbol{w}_i^T\boldsymbol{b}$, the more the mode can be controlled by u
- the larger $(\boldsymbol{c}^T\boldsymbol{V})_i = \boldsymbol{c}^T\boldsymbol{v}_i$, the more the mode can be observed in z.

Transfer function and residues

$$F(s) = \frac{Z(s)}{U(s)} = \boldsymbol{c}^T \boldsymbol{V} (s\boldsymbol{I} - \boldsymbol{\Lambda})^{-1} \boldsymbol{W}\boldsymbol{b} + d$$

$$= \begin{bmatrix} \boldsymbol{c}^T \boldsymbol{v}_1 & \dots & \boldsymbol{c}^T \boldsymbol{v}_n \end{bmatrix} diag \left(\frac{1}{s - \lambda_i} \right) \begin{bmatrix} \boldsymbol{w}_1^T \boldsymbol{b} \\ \vdots \\ \boldsymbol{w}_n^T \boldsymbol{b} \end{bmatrix} + d$$

$$= \sum_{i=1}^{n} \frac{\boldsymbol{c}^T \boldsymbol{v}_i \boldsymbol{w}_i^T \boldsymbol{b}}{s - \lambda_i} + d = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} + d$$

The *residue* R_i relative to the i-th mode λ_i :

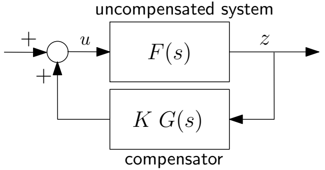
- depends on both the observability and the controllability of λ_i
- is even smaller than $F(s)$ has a zero ζ_k close to λ_i. Indeed:

$$R_i = \lim_{s \to \lambda_i} (s - \lambda_i) F(s) = \lim_{s \to \lambda_i} (s - \lambda_i) \frac{\prod_{k=1}^{m} (s - \zeta_k)}{\prod_{j=1}^{n} (s - \lambda_j)} = \lim_{s \to \lambda_i} \frac{\prod_{k=1}^{m} (s - \zeta_k)}{\prod_{j=1, j \neq i}^{n} (s - \lambda_j)}$$

- would be zero in case of exact *zero-pole cancellation*.

Synthesis of a stabilizing feedback using residues

Consider a compensator
using z as input and acting on u



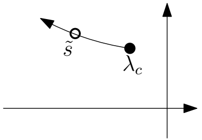
Which condition should be satisfied by the transfer function $G(s)$ in order to stabilize the mode λ_c of the uncompensated system ?

The closed-loop transfer function is $\dfrac{F(s)}{1 - KF(s)G(s)}$

Let $\tilde{s}$ be one of the closed-loop poles:

$$\begin{aligned} 1 - KF(\tilde{s})G(\tilde{s}) &= 0 \\ 1 - K\left[\sum_i \frac{R_i}{\tilde{s}-\lambda_i} + d\right] G(\tilde{s}) &= 0 \\ 1 - K\sum_{i\neq c} \frac{R_i}{\tilde{s}-\lambda_i} G(\tilde{s}) - K\frac{R_c}{\tilde{s}-\lambda_c} G(\tilde{s}) - KdG(\tilde{s}) &= 0 \qquad (30) \end{aligned}$$

Consider a closed-loop pole $\tilde{s}$ lying on the branch of the root locus which starts from the open-loop pole λ_c.



When the compensator gain K tends to zero, $\tilde{s}$ tends to λ_c.

Keeping the dominant terms only in (30):

$$1 - R_c\, G(\lambda_c) \lim_{K \to 0} \frac{K}{\tilde{s} - \lambda_c} = 0 \qquad \text{or} \qquad \lim_{K \to 0} \frac{\tilde{s} - \lambda_c}{K} = R_c\, G(\lambda_c)$$

In the complex plane $\lim_{K \to 0} \frac{\tilde{s} - \lambda_c}{K}$ is a vector tangent to the branch of the root locus starting from λ_c.

In order to shift the eigenvalue λ_c to the left :

- the branch of the root locus should leave λ_c at an angle of 180 degrees
- $R_c\, G(\lambda_c)$ should be a real negative number
- $G(s)$ must be such that $\angle G(\lambda_c) = \pm 180^o - \angle R_c$

Improvement of small-disturbance angle stability

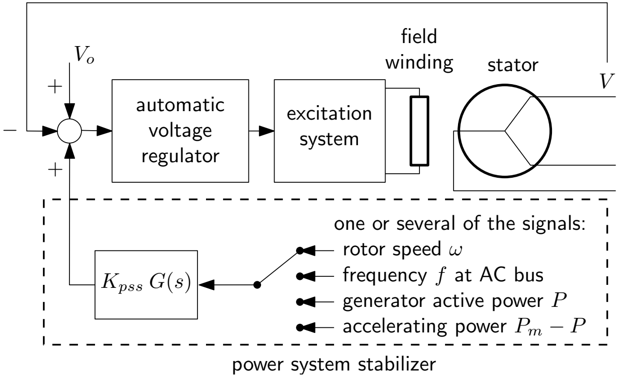
Principle :

- increase the damping torques of synchronous machines
- move the complex eigenvalues corresponding to the unstable or badly damped mode into the desired region of the complex plane.

How and where ?

- add a *power system stabilizer* acting through the Automatic Voltage Regulator (AVR): the less expensive solution
- take advantage of the presence of power electronics-base components to vary
  - the shunt susceptance of a Static Var Compensator (SVC)
  - the active power flowing through a High Voltage Direct Current (HVDC) link
  - the series reactance of a Thyristor Controlled Series Capacitor (TCSC)
  - another Flexible AC Transmission System (FACTS) device.

Power System Stabilizers (PSS)



Let λ_c be the badly-damped/unstable electromechanical mode.

Since λ_c is close to the imaginary axis: $\lambda_c \simeq j\; imag(\lambda_c) = j\,\omega_c$

The PSS increases the damping torque in a range of frequencies around ω_c.

The PSS transfer function decomposes into:

$$K_{pss}\, G(s) = K_{pss}\, G_1(s)\, G_2(s)\, G_3(s)$$

Transfer function $G_1(s)$:

- shifts λ_c to the left in the complex plane by bringing a phase compensation according to the residue method :

$$\angle G_1(\lambda_c) \simeq \angle G_1(j\,\omega_c) = \pm\, 180^o - \angle R_c$$

- $G_1(s)$ corresponds to one or several *lead-lag filters*: see slide # 35
- the latter are "tuned" to provide their maximum phase shift ϕ_m at the frequency ω_c

Transfer function $G_2(s)$:

- in steady state and for slow variations, the PSS must not affect voltage regulation
- $G_2(s)$ is a *washout* (or high-pass) filter: see slide # 36
- T_w is taken large enough to not modify the phase angle of G_1 for frequencies around ω_c. For instance:

$$\frac{10}{T_w} \simeq \frac{\omega_c}{10}$$

Transfer function $G_3(s)$ (optional) :

- in a thermal power plant, the turbine stages, the generator and the exciter are mounted on a relatively long shaft. The latter has torsional oscillation frequencies in the range $10 - 15$ Hz and higher
- the PSS must not excite those frequencies
- the risk is higher for a PSS using the rotor speed as input signal
- in this case, G_3 is a low-pass filter so that the PSS contribution is negligible at the lowest torsional frequency and above.

Gain K_{pss} :

- adjusted until the corrected mode $\tilde{\lambda}_c$ has a *damping ratio* :

$$\xi = \frac{-real(\tilde{\lambda}_c)}{|\tilde{\lambda}_c|} = \frac{-real(\tilde{\lambda}_c)}{\sqrt{[real(\tilde{\lambda}_c)]^2 + [imag(\tilde{\lambda}_c)]^2}}$$

  higher than some value :

$$\xi \geq 0.05 - 0.10$$

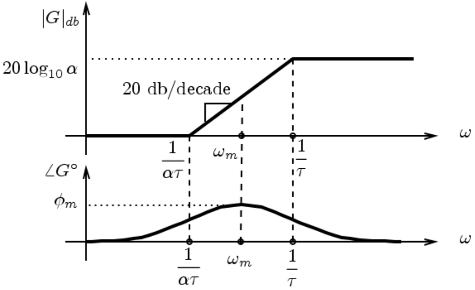
- while K_{pss} is increased, the other eigenvalues are monitored since they might move to the right (the residue method allows controlling a single mode !)
- for excessive values of K_{pss}, the branch of the root locus that starts from λ_c might “bend” to the right (the residue method focuses on a neighbourhood of the mode to correct !)

Lead-lag filter

$$G(s) = \frac{1 + s\alpha\tau}{1 + s\tau}$$

$\alpha > 1$ to obtain phase lead $\qquad$ $\alpha < 1$ to obtain phase lag

Bode plot
(lead filter)



Angular frequency at which the phase is maximum: $\omega_m = \dfrac{1}{\tau\sqrt{\alpha}}$

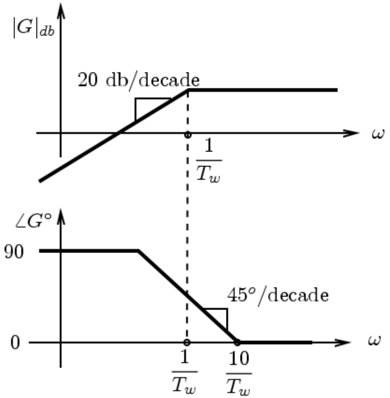
Maximum phase: $\phi_m = \arcsin \dfrac{\alpha - 1}{\alpha + 1} \quad \Rightarrow \quad \alpha = \dfrac{1 + \sin \phi_m}{1 - \sin \phi_m}$

To obtain $\phi_m > 60^o$ use two filters in cascade, etc.

Washout filter

$$G(s) = \frac{s}{1 + sT_w}$$

Bode plot



The phase is negligible for $\omega > \dfrac{10}{T_w}$