ELEC0047 - Power system dynamics, control and stability

The phasor approximation explained with an example

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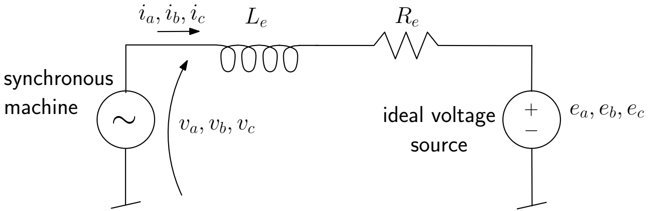
System modelling

Refer to lecture "Behaviour of synchronous machine during a short-circuit (a simple example of electromagnetic transients)".



Network :

- resistance R_e and inductance L_e in each phase
- no magnetic coupling between phases, for simplicity.

$$e_a = \sqrt{2}E\cos(\omega_N t + \theta_e) \quad e_b = \sqrt{2}E\cos(\omega_N t + \theta_e - \frac{2\pi}{3}) \quad e_c = \dots$$

Machine :

- field winding f in the d axis
- one damper winding $q1$ in the q axis
- constant rotor speed : $\theta_r = \theta_r^o + \omega_N t$
- constant excitation voltage V_f.

The phasor approximation

. . . is a simplification of the power system model regarding the network and the components connected to it. It relies on the following :

Assumption 1. In the network the voltage and current evolutions take on the form:

$$v(t) = \sqrt{2}\, V(t) \cos(\omega_N t + \theta(t)) \quad (1)$$

$$i(t) = \sqrt{2}\, I(t) \cos(\omega_N t + \psi(t)) \quad (2)$$

where the effective values and the phase angles vary with time.

Eqs. (1, 2) can be rewritten as:

$$v(t) = \sqrt{2}\, re\left[V(t)e^{j\theta(t)}e^{j\omega_N t}\right] = \sqrt{2}\, re\left[(v_x(t) + j\, v_y(t))\, e^{j\omega_N t}\right] \quad (3)$$

$$i(t) = \sqrt{2}\, re\left[I(t)e^{j\psi(t)}e^{j\omega_N t}\right] = \sqrt{2}\, re\left[(i_x(t) + j\, i_y(t))\, e^{j\omega_N t}\right] \quad (4)$$

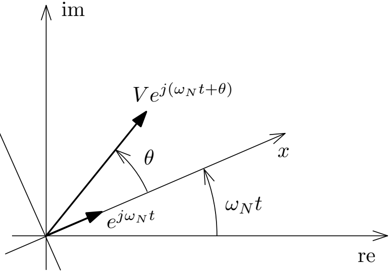
Introducing the time-varying phasors

Reminder. Consider the sinusoidal evolution :

$$v_a(t) = \sqrt{2} V \cos(\omega_N t + \theta) = \sqrt{2}\, re \left[V\, e^{j(\omega_N t + \theta)} \right] = \sqrt{2}\, re \left[V\, e^{j\theta}\, e^{j\omega_N t} \right]$$

The phasor $V\,e^{j\theta}$ has two interpretations :

1. it coincides with the rotating vector $V\,e^{j(\omega_N t+\theta)}$ at time $t = 0$
2. it is, at any time, the rotating vector $V\,e^{j(\omega_N t+\theta)}$ expressed with respect to (x, y) axes rotating at the angular speed ω_N.



Similarly (and by extension) the phasor :

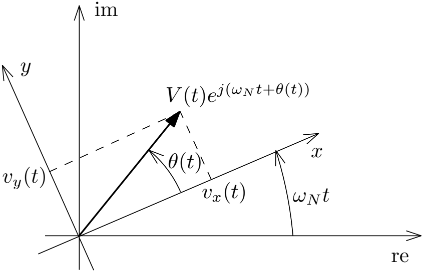
$$V(t)e^{j\theta(t)} = v_x(t) + j\, v_y(t)$$

is the rotating vector :

$$V(t)e^{j\theta(t)}e^{j\omega_N t} = (v_x(t) + j\, v_y(t))\, e^{j\omega_N t}$$

expressed with respect to (x, y) axes rotating at the angular speed ω_N.

The corresponding phasor diagram :



Remark. For the voltage source $e_a(t) = \sqrt{2}E\cos(\omega_N t + \theta_e)$, the phasor is constant:

$$\bar{E}_a = E\, e^{j\theta_e} = e_x + je_y$$

Transformation of the network equations

The network equation relative to phase a is:

$$v_a - e_a = R_e i_a + L_e \frac{di_a}{dt}$$

Replacing the voltage and the current by their expressions (3, 4) :

$$\sqrt{2}\, re\left[(v_x(t) + j\, v_y(t))\, e^{j\omega_N t}\right] - \sqrt{2}\, re\left[(e_x + j\, e_y)\, e^{j\omega_N t}\right]$$
$$= R_e \sqrt{2}\, re\left[(i_x(t) + j\, i_y(t))\, e^{j\omega_N t}\right] + L_e \frac{d}{dt}\left\{\sqrt{2}\, re\left[(i_x(t) + j\, i_y(t))\, e^{j\omega_N t}\right]\right\}$$

Passing to complex numbers and dividing by $\sqrt{2}$:

$$(v_x(t) + j\, v_y(t))\, e^{j\omega_N t} - (e_x + j\, e_y)\, e^{j\omega_N t} = R_e (i_x(t) + j\, i_y(t))\, e^{j\omega_N t}$$
$$+ L_e \frac{d}{dt}\left\{(i_x(t) + j\, i_y(t))\, e^{j\omega_N t}\right\}$$

Developing the derivative :

$$(v_x(t) + j\, v_y(t))\, e^{j\omega_N t} - (e_x + j\, e_y)\, e^{j\omega_N t} = R_e(i_x(t) + j\, i_y(t))\, e^{j\omega_N t}$$
$$+ L_e \frac{d}{dt}(i_x(t) + j\, i_y(t)) e^{j\omega_N t} + j\omega_N L_e (i_x(t) + j\, i_y(t)) e^{j\omega_N t}$$

Dividing by $e^{j\omega_N t}$:

$$v_x(t) + j\, v_y(t) - e_x - j\, e_y = R_e(i_x(t) + j\, i_y(t))$$
$$+ L_e \frac{d}{dt}(i_x(t) + j\, i_y(t)) + j\omega_N L_e (i_x(t) + j\, i_y(t))$$

Decomposing into real and imaginary components:

$$v_x(t) - e_x = R_e i_x(t) + \frac{d}{dt} L_e i_x(t) - \omega_N L_e i_y(t) \quad (5)$$
$$v_y(t) - e_y = R_e i_y(t) + \frac{d}{dt} L_e i_y(t) + \omega_N L_e i_x(t) \quad (6)$$

Assumption 2a. The terms $\frac{d}{dt} L_e i_x(t)$ and $\frac{d}{dt} L_e i_y(t)$, i.e. the rate of change of fluxes in network inductances, are neglected. This leads to neglecting some short-lasting components of the current.

Eqs. (5, 6) become:

$$\begin{aligned} v_x(t) - e_x &= R_e i_x(t) - X_e i_y(t) && (7) \\ v_y(t) - e_y &= R_e i_y(t) + X_e i_x(t) && (8) \end{aligned}$$

where $X_e = \omega_N L_e$ is the reactance of the line.

Let us define the time-varying phasors referred to the (x, y) axes:

$$\bar{V}(t) = v_x(t) + j\, v_y(t) \qquad \bar{I}(t) = i_x(t) + j\, i_y(t)$$

Eqs. (7, 8) can be recombined into the single complex relation:

$$\bar{V}(t) - \bar{E} = R_e \bar{I}(t) + jX_e \bar{I}(t)$$

which is the equation of the (R_e, L_e) circuit in sinusoidal steady state, *but with the time-varying phasors* $\bar{V}$ *and* $\bar{I}$.

Assumption 2b. The same simplification is made in the machine stator model, i.e. the transformer voltages are neglected.

The Park equations of the stator become:

$$v_d = -R_a i_d - \psi_q - \cancel{\frac{d\psi_d}{dt}} \tag{9}$$

$$v_q = -R_a i_q + \psi_d - \cancel{\frac{d\psi_q}{dt}} \tag{10}$$

The other machine equations are unchanged:

$$\psi_d = L_{dd} i_d + L_{df} i_f \tag{11}$$

$$\psi_q = L_{qq} i_q + L_{qq1} i_{q1} \tag{12}$$

$$\psi_f = L_{ff} i_f + L_{df} i_d \tag{13}$$

$$\psi_{q1} = L_{qq1} i_q + L_{q1q1} i_{q1} \tag{14}$$

$$\frac{1}{\omega_N}\frac{d}{dt}\psi_f = V_f - R_f i_f \tag{15}$$

$$\frac{1}{\omega_N}\frac{d}{dt}\psi_{q1} = -R_{q1} i_{q1} \tag{16}$$

Assumption 3. The operation is three-phase balanced.

Thus, the voltages and currents take on the form:

$$v_a(t) = \sqrt{2}V(t)\cos(\omega_N t + \theta(t)) \qquad i_a(t) = \sqrt{2}I(t)\cos(\omega_N t + \psi(t))$$
$$v_b(t) = \sqrt{2}V(t)\cos(\omega_N t + \theta(t) - \frac{2\pi}{3}) \qquad i_b(t) = \sqrt{2}I(t)\cos(\omega_N t + \psi(t) - \frac{2\pi}{3})$$
$$v_c(t) = \sqrt{2}V(t)\cos(\omega_N t + \theta(t) - \frac{4\pi}{3}) \qquad i_c(t) = \sqrt{2}I(t)\cos(\omega_N t + \psi(t) - \frac{4\pi}{3})$$

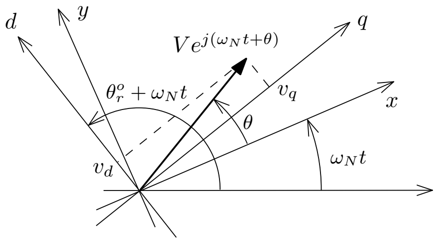
Applying the Park transformation and passing in per unit:

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = \frac{1}{\sqrt{3}V_B}\mathcal{P}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} V(t)\ \cos(\theta_r^o - \theta(t)) \\ V(t)\ \sin(\theta_r^o - \theta(t)) \\ 0 \end{bmatrix} \qquad (17)$$

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \frac{1}{\sqrt{3}I_B}\mathcal{P}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} I(t)\ \cos(\theta_r^o - \psi(t)) \\ I(t)\ \sin(\theta_r^o - \psi(t)) \\ 0 \end{bmatrix} \qquad (18)$$

These results are the same as for the machine in steady-state operation, except that V, I, θ and ψ vary with time.

Phasors in the (q, d) and (x, y) reference frames



As for the machine in steady state, $(v_d,\ v_q)$ are the projections on the (d,q) axes of the rotating vector $Ve^{j\omega_N t+\theta}$:

- projection on the q axis:

$$V\cos\left[\omega_N t + \theta - (\theta_r^o + \omega_N t - \frac{\pi}{2})\right] = V_a\cos(\theta - \theta_r^o + \frac{\pi}{2}) = V\sin(\theta_r^o - \theta) = v_q$$

- projection on the d axis:

$$V\sin\left[\omega_N t + \theta - (\theta_r^o + \omega_N t - \frac{\pi}{2})\right] = V_a\sin(\theta - \theta_r^o + \frac{\pi}{2}) = V_a\cos(\theta_r^o - \theta) = v_d$$

Similarly for the current phasor.

Relation between v_d, v_q and v_x, v_y ?

$$\begin{aligned} v_d &= V\cos(\theta_r^o - \theta) = V(\cos\theta_r^o \cos\theta + \sin\theta_r^o \sin\theta) \\ &= \cos\theta_r^o \; V\cos\theta + \sin\theta_r^o \; V\sin\theta = \cos\theta_r^o \; v_x + \sin\theta_r^o \; v_y \end{aligned} \quad (19)$$

$$\begin{aligned} v_q &= V\sin(\theta_r^o - \theta) = V(\sin\theta_r^o \cos\theta - \cos\theta_r^o \sin\theta) \\ &= \sin\theta_r^o \; V\cos\theta - \cos\theta_r^o \; V\sin\theta = \sin\theta_r^o \; v_x - \cos\theta_r^o \; v_y \end{aligned} \quad (20)$$

Similarly for the currents:

$$i_d = \cos\theta_r^o \; i_x + \sin\theta_r^o \; i_y \quad (21)$$

$$i_q = \sin\theta_r^o \; i_x - \cos\theta_r^o \; i_y \quad (22)$$

The whole (simplified) model

$$
\begin{aligned}
\text{rotor windings :} \quad \frac{1}{\omega_N}\frac{d\psi_f}{dt} &= v_f - R_f i_f \\
\frac{1}{\omega_N}\frac{d\psi_{q1}}{dt} &= -R_{q1} i_{q1} \\
\text{flux-current :} \quad 0 &= \psi_d - L_{dd} i_d - L_{df} i_f \\
0 &= \psi_q - L_{qq} i_q - L_{qq1} i_{q1} \\
0 &= \psi_f - L_{ff} i_f - L_{df} i_d \\
0 &= \psi_{q1} - L_{qq1} i_q - L_{q1q1} i_{q1} \\
\text{stator Park :} \quad 0 &= v_d + R_a i_d + \psi_q \\
0 &= v_q + R_a i_q - \psi_d \\
\text{network :} \quad 0 &= v_x - e_x - R_e i_x + X_e i_y \\
0 &= v_y - e_y - R_e i_y - X_e i_x \\
(d,q) \leftrightarrow (x,y) : \quad 0 &= v_d - \cos\theta_r^o \; v_x - \sin\theta_r^o \; v_y \\
0 &= v_q - \sin\theta_r^o \; v_x + \cos\theta_r^o \; v_y \\
0 &= i_d - \cos\theta_r^o \; i_x - \sin\theta_r^o \; i_y \\
0 &= i_q - \sin\theta_r^o \; i_x + \cos\theta_r^o \; i_y
\end{aligned}
$$

Numerical example

- Same data as in the lecture "Behaviour of synchronous machine during a short-circuit"
- specified from reactances and time constants

Network and machine data

$f_N = 50$ Hz

$X_e = L_e = 0.20$ pu $\quad R_e = 0.01$ pu

$Ra = 0.005$ pu

$X_d = L_{dd} = 2.4$ pu $\quad X_q = L_{qq} = 2.4$ pu

$X_\ell = L_\ell = L_{dd} - L_{df} = L_{qq} - L_{qq1} = 0.2$ pu

$X'_d = L'_d = 0.4$ pu $\quad X''_q = L''_q = 0.25$ pu

$T'_{do} = L_{ff}/R_f = 7.0$ s $\quad T''_{qo} = L_{qq1}/R_{q1} = 0.3$ s

(L_{oo} not needed)

Initial operating point

$P = 0.5$ pu

$Q = 0.1$ pu

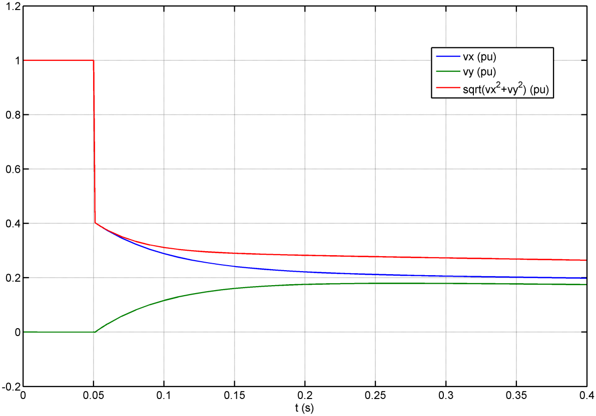
$\bar{V}_a = 1.000$ pu $\angle 0$

Simulation results

A MATLAB script to simulate this system is available in `phasormode.m` and `matA.m`.

A three-phase short-circuit is simulated by setting E to zero at $t = 0.05$ s.

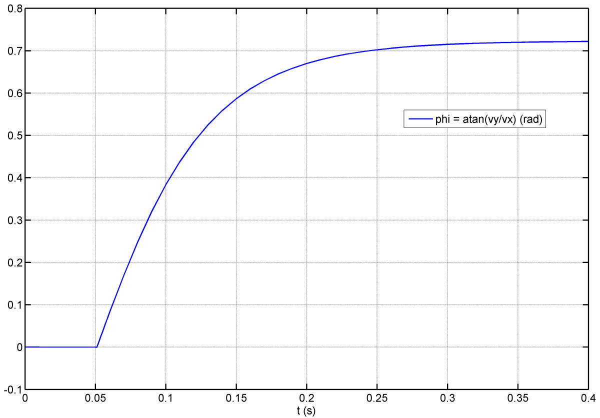
The following plots should be compared with the corresponding curves in the lecture “Behaviour of synchronous machine during a short-circuit”



- effective (RMS) value of the phase voltage :

$$V = \sqrt{v_x^2 + v_y^2} = \sqrt{v_d^2 + v_q^2}$$

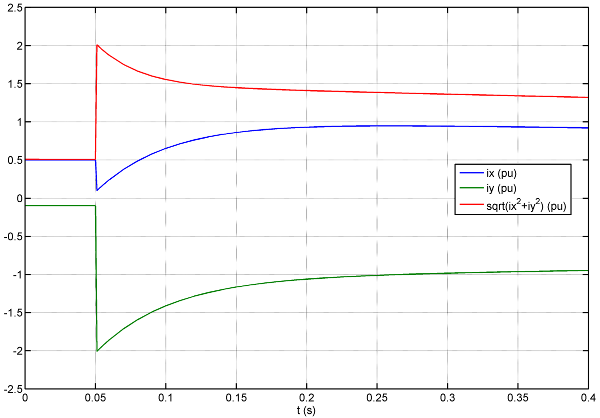
- note that v_x, v_y, v_d and v_q are constant in steady state, although the corresponding voltage $v_a(t)$ evolves sinusoidally



- phase angle of the voltage of phase a:

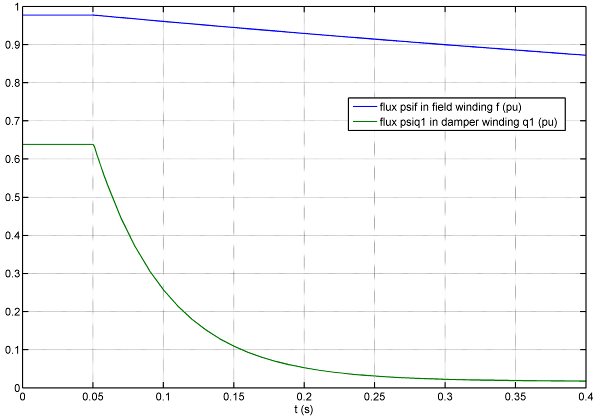
$$\theta = atan\left(\frac{v_y}{v_x}\right)$$

- both θ and V evolve with time

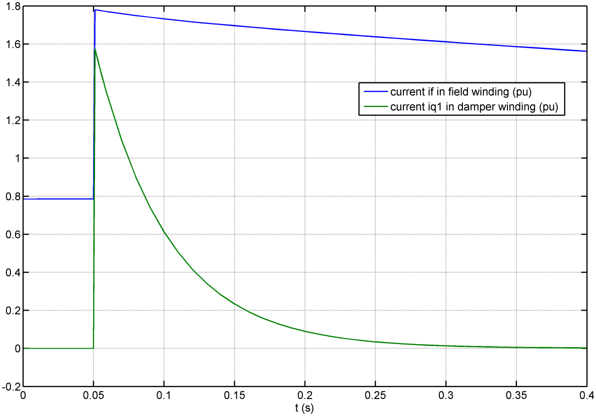


- effective (RMS) value of the phase current:

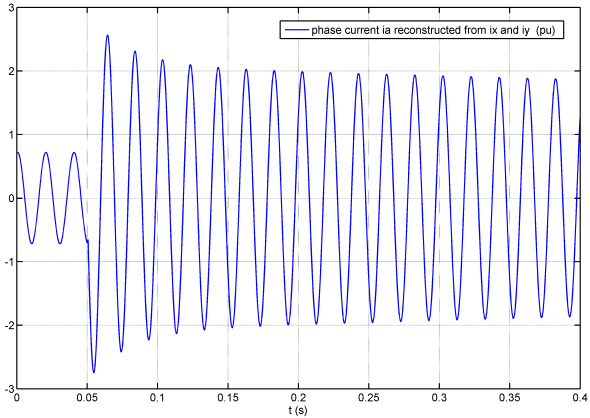
$$I = \sqrt{i_x^2 + i_y^2} = \sqrt{i_d^2 + i_q^2}$$



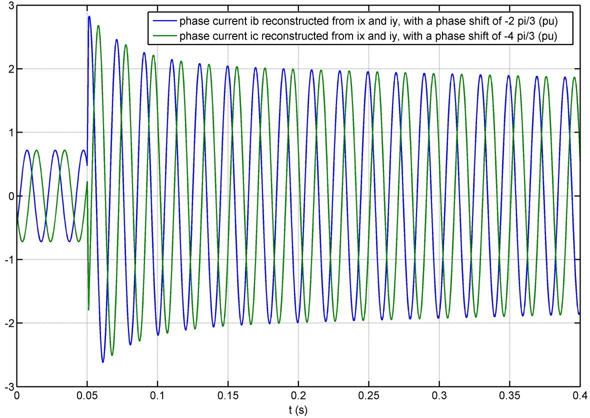
- the simulation in phasor mode renders the aperiodic evolution of each flux, but not its oscillatory component (stemming from the magnetic field H_{DC})



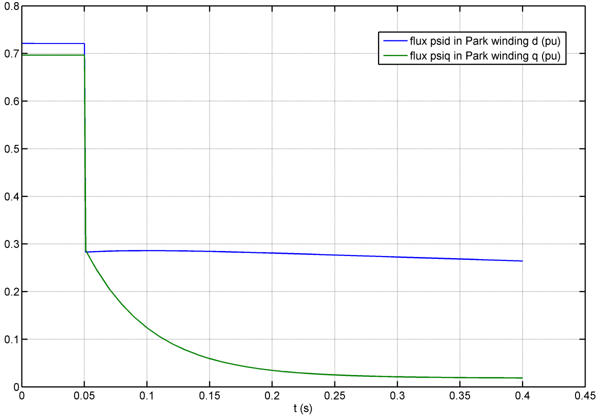
- similar remark for the currents in rotor windings
- the various curves show that the variables evolve much more smoothly than in the electromagnetic transient simulation
- hence, a much larger time step can be used in numerical simulation (e.g. 1/4 to 1 cycle at fundamental frequency f_N)
- simulations can be run over much longer times (e.g. up to 10-15 minutes)



- the current $i_a(t)$ has been "reconstructed" from its components i_x and i_y
- note, however, that the simulation in phasor mode is not used to obtain the "full wave" evolution of voltages or currents
- instead, it provides the evolution of the associated phasors



- the currents $i_b(t)$ and $i_c(t)$ have been "reconstructed" from i_x and i_y with a phase shift of $\pm 2\pi/3$ from one phase to the other
- due to the terms neglected in the electromagnetic transient ("full") model:
  - the simulation in phasor mode neglects the aperiodic components of currents
  - the currents undergo "non-physical" discontinuities



- due to the terms neglected in the electromagnetic transient ("full") model, the fluxes in the d and q windings undergo "non-physical" discontinuities, as for the stator currents