*ELEC0047 - Power system dynamics, control and stability*

# Power system modelling under the phasor approximation

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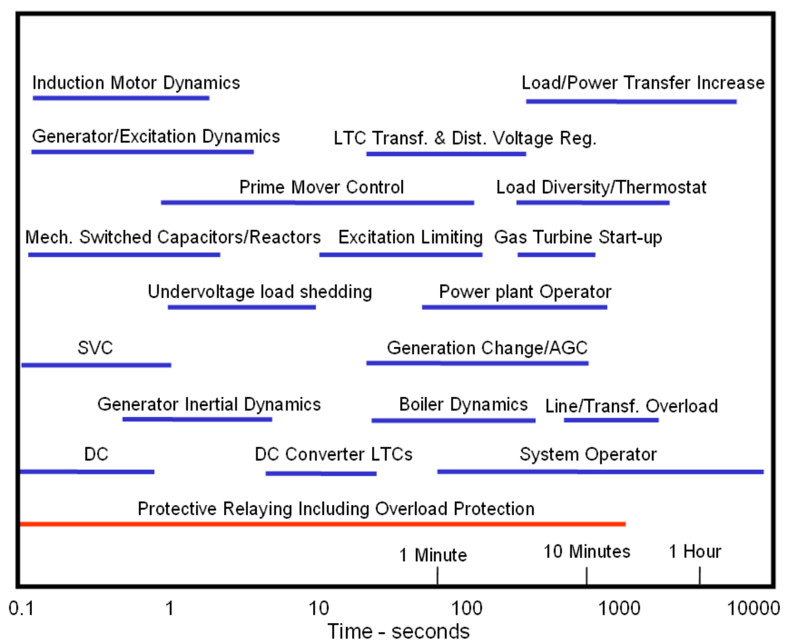
# Electromagnetic transient vs. phasor-mode simulations

| **electromagnetic transient simul.** | **simul. under phasor approximation** |
|---|---|
| $d\psi/dt$ terms included | $d\psi/dt$ terms neglected<br>(in network and connected equipment) |
| virtually any dynamics included | sinusoidal evolution with<br>varying magnitude and phase angle;<br>$\Rightarrow$ ignore dynamics shorter than $\simeq$ 1 cycle |
| computes the "full-wave" evolutions<br>of voltages and currents | computes the evolutions<br>of magnitudes and phase angles<br>of voltages and current phasors |
| time step smaller than $\simeq$ 0.0005 s | time step larger than 1/4 cycle<br>(except after a discontinuity) |
| network modeled with differential eqs.<br>of inductors and capacitors | network modeled by admittance matrix<br>and voltage/current phasors |

| **electromagnetic transient simul.** | **simul. under phasor approximation** |
|---|---|
| three-phase representation | single-phase representation |
| all imbalanced conditions | by default, balanced conditions<br>(+ corrections for inverse and zero sequences) |
| not suitable for large-scale studies | suitable for large-scale studies<br>(e.g. 25000 buses for the continental European grid) |
| simulated time up to $\simeq$ 10 s | simulated time<br>from a few seconds to 10-15 minutes |
| used to design components<br>+ "hardware-in-the-loop" | used for system-wide stability studies<br>(angle, frequency, voltage) |
| *Examples:*<br>EMTP-RV, PSCAD, RTDS, ... | *Examples:*<br>Eurostag, Power Factory, PSS/E, ...<br>*RAMSES* |

# Dynamics considered in phasor-mode simulation



# Network modelling under the phasor approximation

## Network equations based on bus admittance matrix

$$\bar{\mathbf{I}} = \mathbf{Y}\bar{\mathbf{V}} \tag{1}$$

$\bar{\mathbf{I}}$ : vector of complex currents injected into the network at the various nodes
$\bar{\mathbf{V}}$ : vector of complex voltages at the various buses
$\mathbf{Y}$ : bus (or nodal) admittance matrix

Which frequency consider in the admittances ?

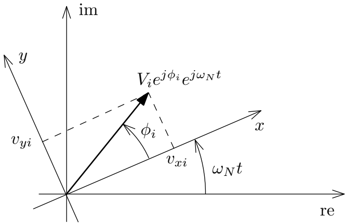
- In dynamic regime, each synchronous machine defines a local frequency
- in most cases those various frequencies remain close to $f_N$
- for large deviations with respect to the nominal value $f_N$, the admittances entering the $\bar{\mathbf{Y}}$ matrix can be updated with the average system frequency
- otherwise, the admittances are simply computed at frequency $f_N$
- this is consistent with the approximation $\omega \simeq 1$ pu made in the Park equations of the synchronous machines.

## The synchronous reference frame

Under the phasor approximation, the voltage at the $i$-th bus[1] takes on the form:

$$
\begin{aligned}
v_i(t) &= \sqrt{2} V_i(t) \cos(\omega_N t + \phi_i(t)) = \sqrt{2} \; re \left[ V_i(t) e^{j\,\phi_i(t)} e^{j\,\omega_N t} \right] \\
&= \sqrt{2} \; re \left[ \left( v_{xi}(t) + j v_{yi}(t) \right) e^{j\,\omega_N t} \right]
\end{aligned}
$$

$v_{xi}(t) + j\, v_{yi}(t)$ is the voltage phasor, in rectangular coordinates, expressed with respect to $(x, y)$ axes rotating at the angular speed $\omega_N$.




These $(x, y)$ axes are said to make up a *synchronous reference frame*.

[1]this is just an example. Similar expressions hold true for currents

Main limitation of the synchronous reference frame :

- after a disturbance the system settles at a new frequency $f \neq f_N$
- all phasors rotate at the angular speed $2\pi f \neq \omega_N$
- phasor components such as $v_{xi}$ and $v_{yi}$ oscillate at frequency $|f - f_N|$, although the system is at equilibrium from a practical viewpoint.

- The synchronous reference is not suitable for long-term simulation, since tracking the above oscillations requires using a small enough time step size
- it is thus used in short-term simulation (where frequency has not yet returned to steady state).

In fact, any reference frame $(x, y)$ can be used

- it does not need to rotate at the angular speed $\omega_N$; it can rotate at any known speed which is convenient
- the only constraint is that all voltage and current phasors must refer to the same axes.

## The center of inertia reference

Consider axes $(x, y)$ rotating at the angular speed :

$$\dot{\theta}_{coi} = \frac{d}{dt}\left(\frac{\sum_{i=1}^{m} M_i \theta_{ri}}{\sum_{i=1}^{m} M_i}\right) = \frac{\sum_{i=1}^{m} M_i \dot{\theta}_{ri}}{\sum_{i=1}^{m} M_i}$$

where:

$m$ is the total number of synchronous machines

$\dot{\theta}_{ri}$ is the angular speed of $i$-th synchronous machine $(i = 1, \dots, m)$ (rad/s)

$M_i$ is the inertia of the $i$-th machine expressed on a common base power $S_B$

$$M_i = 2H_i \frac{S_{Ni}}{S_B}$$

$\dot{\theta}_{coi}$ is the angular speed of the *center of inertia* (rad/s).

- When the system settles at a frequency $f$, all synchronous machines rotate at the speed $2\pi f$, and so do the reference axes $(\omega_{coi} = 2\pi f)$
- phasor components such as $v_{xi}$ and $v_{yi}$ tend to constant values
- simulation is less demanding and a larger time step size can be used
- this reference frame is suitable for long-term simulation.

# Network equations in an $(x, y)$ reference frame

With all voltage and current phasors referred to the $(x, y)$ axes, the network equations take on the form:

$$\mathbf{i}_x + j\,\mathbf{i}_y = \mathbf{Y}\,(\mathbf{v}_x + j\,\mathbf{v}_y) = (\mathbf{G} + j\,\mathbf{B})\,(\mathbf{v}_x + j\,\mathbf{v}_y)$$

with:

$$\mathbf{v}_x = \begin{bmatrix} v_{x1} \\ \vdots \\ v_{xN} \end{bmatrix} \quad \mathbf{v}_y = \begin{bmatrix} v_{y1} \\ \vdots \\ v_{yN} \end{bmatrix} \quad \mathbf{i}_x = \begin{bmatrix} i_{x1} \\ \vdots \\ i_{xN} \end{bmatrix} \quad \mathbf{i}_y = \begin{bmatrix} i_{y1} \\ \vdots \\ i_{yN} \end{bmatrix}$$

$\mathbf{G}$: conductance matrix $\qquad$ $\mathbf{B}$: susceptance matrix

Decomposing into real and imaginary parts and assembling into a single equation:

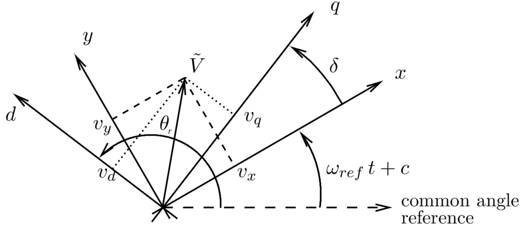
$$\begin{bmatrix} \mathbf{i}_x \\ \mathbf{i}_y \end{bmatrix} = \begin{bmatrix} \mathbf{G} & -\mathbf{B} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{v}_x \\ \mathbf{v}_y \end{bmatrix}$$

$N$ buses $\longrightarrow$ $2N$ equations involving $4N$ variables

# Incorporating synchronous machines

## Passing from individual machine to network reference frame

The $(d, q)$ reference frame simplifies a lot the machine model but it is a "local" reference. It is required to get back to the common reference.




The reference axes $(x, y)$ rotate at the angular speed $\omega_{ref}$

$c$ is an arbitrary constant

$\delta$ is often called *rotor angle*

$$\tilde{V} = (v_x + jv_y)e^{j(\omega_{ref} t + c)} = (v_q + jv_d)e^{j(\delta + \omega_{ref} t + c)}$$

$$\Leftrightarrow \quad v_x + jv_y = (v_q + jv_d)e^{j\delta} = (v_q + jv_d)(\cos\delta + j\sin\delta)$$

Decomposing into real and imaginary parts:

$$v_d = -\sin\delta\, v_x + \cos\delta\, v_y$$

$$v_q = \cos\delta\, v_x + \sin\delta\, v_y$$

Similarly for the currents:

$$\begin{aligned} i_d &= -\sin\delta\ i_x + \cos\delta\ i_y \\ i_q &= \cos\delta\ i_x + \sin\delta\ i_y \end{aligned}$$

The machine stator equations[2]:

$$\begin{aligned} v_d &= -R_a i_d - \psi_q = -R_a i_d - (L_\ell i_q + \psi_{aq}) \\ v_q &= -R_a i_q + \psi_d = -R_a i_q + (L_\ell i_d + \psi_{ad}) \end{aligned}$$

can be expressed in terms of $v_x, v_y, i_x, i_y$ as follows:

$$\begin{aligned} (-\sin\delta\ v_x + \cos\delta\ v_y) + R_a(-\sin\delta\ i_x + \cos\delta\ i_y) + L_\ell(\cos\delta\ i_x + \sin\delta\ i_y) + \psi_{aq} &= 0 \\ (\cos\delta\ v_x + \sin\delta\ v_y) + R_a(\cos\delta\ i_x + \sin\delta\ i_y) - L_\ell(-\sin\delta\ i_x + \cos\delta\ i_y) - \psi_{ad} &= 0 \end{aligned}$$

[2] See lecture "Dynamics of the synchronous machine" - Section "Model simplifications"

**Adjusting the motion equation** to change from the variable $\theta_r$ to the variable $\delta$

$$\theta_r = \omega_{ref} t + c + \delta + \frac{\pi}{2} \quad \Leftrightarrow \quad \frac{d}{dt}\theta_r = \omega_{ref} + \frac{d}{dt}\delta$$

Hence, the previous equation :

$$\frac{1}{\omega_N}\frac{d}{dt}\theta_r = \omega$$

is replaced by :

$$\frac{1}{\omega_N}\frac{d}{dt}\delta = \omega - \frac{\omega_{ref}}{\omega_N}$$

- with the synchronous reference : $\frac{1}{\omega_N}\frac{d}{dt}\delta = \omega - 1$
- with the COI reference : $\frac{1}{\omega_N}\frac{d}{dt}\delta = \omega - \omega_{coi}$ where $\omega_{coi} = \frac{\sum_{i=1}^m M_i \omega_i}{\sum_{i=1}^m M_i}$

where $\omega$, $\omega_i$ and $\omega_{coi}$ are in per unit.

The motion equation $2H\frac{d}{dt}\omega = T_m - T_e$ is unchanged.

# Incorporating induction machines

The induction machine model relies on $(d, q)$ axes rotating at $\omega_s$, the stator angular speed.

- In order the model to properly account for the effect of frequency, it is appropriate to use the center of inertia reference and approximate $\omega_s$ by $\omega_{coi}$ (the average angular frequency of the whole system)
- make the $q$ axis coincide with the $x$ axis
- make the $d$ axis coincide with the $y$ axis.

The machine stator equations[3]:

$$\begin{aligned} v_{ds} &= R_s i_{ds} + \omega_s \psi_{qs} = R_s i_{ds} + \omega_s L_{ss} i_{qs} + \omega_s L_{sr} i_{qr} \\ v_{qs} &= R_s i_{qs} - \omega_s \psi_{ds} = R_s i_{qs} - \omega_s L_{ss} i_{ds} - \omega_s L_{sr} i_{dr} \end{aligned}$$

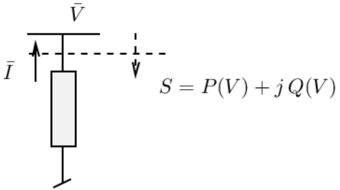
become :

$$\begin{aligned} v_y - R_s i_y - \omega_{coi} L_{ss} i_x - \omega_{coi} L_{sr} i_{qr} &= 0 \\ v_x - R_s i_x + \omega_{coi} L_{ss} i_y + \omega_{coi} L_{sr} i_{dr} &= 0 \end{aligned}$$

[3] See lecture "Dynamics of the induction machine"

# Incorporating static loads

Consider a load whose active and reactive powers vary with the terminal voltage $V$ according to known algebraic relations $P(V)$ and $Q(V)$.




voltage magnitude: $V = \sqrt{v_x^2 + v_y^2}$

Note: in the formulation (1) the bus currents are oriented into the network

$$P(V) + j\,Q(V) = -\bar{V}\bar{I}^{\star} = -(v_x + j\,v_y)(i_x - j\,i_y)$$

Decomposing into real and imaginary parts:

$$\begin{aligned} P\left(\sqrt{v_x^2 + v_y^2}\right) + v_x i_x + v_y i_y &= 0 \\ Q\left(\sqrt{v_x^2 + v_y^2}\right) - v_x i_y + v_y i_x &= 0 \end{aligned}$$

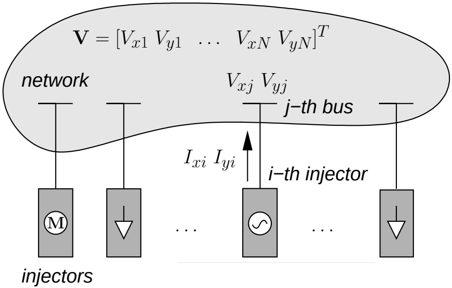
## Extension

Consider a load whose active and reactive powers vary with the terminal voltage $V$ and frequency $f$ according to known algebraic relations $P(V, f)$ and $Q(V, f)$.

How is the frequency evaluated in phasor-mode simulation ?

- during transients, there is not a *single* frequency
- the frequency at a bus can be obtained *numerically* as the derivative of the voltage phase angle at that bus
- instead, the average system frequency $\omega_{coi}$ can be used, as for the induction machine (thus, the same value is used for all loads).

# Overall model of components connected to network




State vector of $i$-th "injector":

$$\mathbf{x}_i = [i_{xi}\ i_{yi} \ldots]^T \tag{2}$$

Differential-algebraic model of $i$-th injector:

$$\boldsymbol{\Gamma}_i\ \dot{\boldsymbol{x}}_i = \boldsymbol{f}_i(\boldsymbol{x}_i, v_{xj}, v_{yj}) \tag{3}$$

with $\dim \boldsymbol{x}_i = \dim \boldsymbol{f}_i$.

$\boldsymbol{\Gamma}_i$ : square matrix with zero elements, except:

$$[\boldsymbol{\Gamma}_i]_{k\ell} = 1 \quad \text{if the } k\text{-th equation gives} \quad [\dot{\boldsymbol{x}}_i]_\ell$$