ELEC0047 - Power system dynamics, control and stability

Dynamics of the synchronous machine

Thierry Van Cutsem
t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

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Time constants and characteristic inductances

Objective

- define accurately a number of time constants and inductances characterizing the machine electromagnetic transients
- use these expressions to derive from measurements the inductances and resistances of the Park model

Assumption

As we focus on *electromagnetic transients*, the rotor speed $\dot{\theta}$ is assumed constant, since it varies much more slowly.

Laplace transform of Park equations

$$\begin{bmatrix} V_d(s) + \dot{\theta}_r \psi_q(s) \\ -V_f(s) \\ 0 \end{bmatrix} = -\underbrace{\begin{bmatrix} R_a + sL_{dd} & sL_{df} & sL_{dd_1} \\ sL_{df} & R_f + sL_{ff} & sL_{fd_1} \\ sL_{dd_1} & sL_{fd_1} & R_{d_1} + sL_{d_1 d_1} \end{bmatrix}}_{\mathbf{R}_d + s\mathbf{L}_d} \begin{bmatrix} I_d(s) \\ I_f(s) \\ I_{d_1}(s) \end{bmatrix}$$

$$+\mathbf{L}_d \begin{bmatrix} i_d(0) \\ i_f(0) \\ i_{d_1}(0) \end{bmatrix}$$

$$\begin{bmatrix} V_q(s) - \dot{\theta}_r \psi_d(s) \\ 0 \\ 0 \end{bmatrix} = -\underbrace{\begin{bmatrix} R_a + sL_{qq} & sL_{qq_1} & sL_{qq_2} \\ sL_{qq_1} & R_{q_1} + sL_{q_1 q_1} & sL_{q_1 q_2} \\ sL_{qq_2} & sL_{q_1 q_2} & R_{q_2} + sL_{q_2 q_2} \end{bmatrix}}_{\mathbf{R}_q + s\mathbf{L}_q} \begin{bmatrix} I_q(s) \\ I_{q_1}(s) \\ I_{q_2}(s) \end{bmatrix}$$

$$+\mathbf{L}_q \begin{bmatrix} i_q(0) \\ i_{q_1}(0) \\ i_{q_2}(0) \end{bmatrix}$$

Time constants and inductances

Eliminating I_f, I_{d_1}, I_{q_1} and I_{q_2} yields:

$$\begin{aligned} V_d(s) + \dot{\theta}_r \psi_q(s) &= -Z_d(s) I_d(s) + sG(s) V_f(s) \\ V_q(s) - \dot{\theta}_r \psi_d(s) &= -Z_q(s) I_q(s) \end{aligned}$$

where :

$$\begin{aligned} Z_d(s) &= R_a + sL_{dd} - \begin{bmatrix} sL_{df} & sL_{dd_1} \end{bmatrix} \begin{bmatrix} R_f + sL_{ff} & sL_{fd_1} \\ sL_{fd_1} & R_{d_1} + sL_{d_1 d_1} \end{bmatrix}^{-1} \begin{bmatrix} sL_{df} \\ sL_{dd_1} \end{bmatrix} \\ &= R_a + s\ell_d(s) \qquad \ell_d(s) : \textit{d-axis operational inductance} \end{aligned}$$

$$\begin{aligned} Z_q(s) &= R_a + sL_{qq} - \begin{bmatrix} sL_{qq_1} & sL_{qq_2} \end{bmatrix} \begin{bmatrix} R_{q_1} + sL_{q_1 q_1} & sL_{q_1 q_2} \\ sL_{q_1 q_2} & R_{q_2} + sL_{q_2 q_2} \end{bmatrix}^{-1} \begin{bmatrix} sL_{qq_1} \\ sL_{qq_2} \end{bmatrix} \\ &= R_a + s\ell_q(s) \qquad \ell_q(s) : \textit{q-axis operational inductance} \end{aligned}$$

Considering the nature of RL circuits, $\ell_d(s)$ and $\ell_q(s)$ can be factorized into:

$$\ell_d(s) = L_{dd}\frac{(1+sT_d^{'})(1+sT_d^{''})}{(1+sT_{d0}^{'})(1+sT_{d0}^{''})} \quad \text{with} \quad 0 < T_d^{''} < T_{d0}^{''} < T_d^{'} < T_{d0}^{'}$$

$$\ell_q(s) = L_{qq}\frac{(1+sT_q^{'})(1+sT_q^{''})}{(1+sT_{q0}^{'})(1+sT_{q0}^{''})} \quad \text{with} \quad 0 < T_q^{''} < T_{q0}^{''} < T_q^{'} < T_{q0}^{'}$$

Limit values:

$$\lim_{s\to 0} \ell_d(s) = L_{dd} \qquad \textit{d-axis synchronous inductance}$$

$$\lim_{s\to\infty} \ell_d(s) = L_d^{''} = L_{dd}\,\frac{T_d^{'}\,T_d^{''}}{T_{d0}^{'}\,T_{d0}^{''}} \qquad \textit{d-axis subtransient inductance}$$

$$\lim_{s\to 0} \ell_q(s) = L_{qq} \qquad \textit{q-axis synchronous inductance}$$

$$\lim_{s\to\infty} \ell_q(s) = L_q^{''} = L_{qq}\,\frac{T_q^{'}\,T_q^{''}}{T_{q0}^{'}\,T_{q0}^{''}} \qquad \textit{q-axis subtransient inductance}$$

Direct derivation of $L_d^{''}$:

$$\begin{array}{ccccc} & \mathbf{R}_d + s\mathbf{L}_d & \overset{\text{elimin. of } f \text{ and } d_1}{\longrightarrow} & R_a + s\ell_d(s) & \\ s \rightarrow \infty & \downarrow & & \downarrow & s \rightarrow \infty \\ & s\mathbf{L}_d & \underset{\text{elimin. of } f \text{ and } d_1}{\longrightarrow} & sL_d^{''} & \end{array}$$

$$\begin{aligned} L_d^{''} &= L_{dd} - \begin{bmatrix} L_{df} & L_{dd_1} \end{bmatrix} \begin{bmatrix} L_{ff} & L_{fd_1} \\ L_{fd_1} & L_{d_1d_1} \end{bmatrix}^{-1} \begin{bmatrix} L_{df} \\ L_{dd_1} \end{bmatrix} \\ &= L_{dd} - \frac{L_{df}^2 L_{d_1d_1} + L_{ff} L_{dd_1}^2 - 2L_{df} L_{fd_1} L_{dd_1}}{L_{ff} L_{d_1d_1} - L_{fd_1}^2} \end{aligned}$$

and similarly for the q axis.

Transient inductances

If damper winding effects are neglected, the operational inductances simplify into :

$$\ell_d(s) = L_{dd}\frac{1+sT_d^{'}}{1+sT_{d0}^{'}} \qquad \ell_q(s) = L_{qq}\frac{1+sT_q^{'}}{1+sT_{q0}^{'}}$$

and the limit values become :

$$\lim_{s\to\infty} \ell_d(s) = L_d^{'} = L_{dd}\,\frac{T_d^{'}}{T_{d0}^{'}} \qquad \textit{d-axis transient inductance}$$

$$\lim_{s\to\infty} \ell_q(s) = L_q^{'} = L_{qq}\,\frac{T_q^{'}}{T_{q0}^{'}} \qquad \textit{q-axis transient inductance}$$

Using the same derivation as for $L_d^{''}$, one easily gets:

$$L_d^{'} = L_{dd} - \frac{L_{df}^2}{L_{ff}} \qquad L_q^{'} = L_{qq} - \frac{L_{qq_1}^2}{L_{q_1q_1}}$$

Typical values

	machine with round rotor (pu)	machine with salient poles (pu)		machine with round rotor (s)	machine with salient poles (s)
L_d	1.5-2.5	0.9-1.5	T'_{d0}	8.0-12.0	3.0-8.0
L_q	1.5-2.5	0.5-1.1	T'_{d}	0.95-1.30	1.0-2.5
L'_d	0.2-0.4	0.3-0.5	T''_{d0}	0.025-0.065	0.025-0.065
L'_q	0.2-0.4		T''_{d}	0.02-0.05	0.02-0.05
L''_d	0.15-0.30	0.25-0.35	T'_{q0}	2.0	
L''_q	0.15-0.30	0.25-0.35	T'_{q}	0.8	
			T''_{q0}	0.20-0.50	0.04-0.15
			T''_{q}	0.02-0.05	0.02-0.05
			T_α	0.02-0.60	0.02-0.20

inductances in per unit on the machine nominal voltage and apparent power

Comments

- in the direct axis: pronounced "time decoupling":

$$T'_{d0} \gg T''_{d0} \qquad T'_{d} \gg T''_{d}$$

  - *subtransient time constants* T''_{d} and T''_{d0}: short, originate from damper winding
  - *transient time constants* T'_{d} and T'_{d0}: long, originate from field winding
- in the quadrature axis: less pronounced time decoupling
  - because the windings are of quite different nature !
- salient-pole machines: single winding in q axis $\Rightarrow$ the parameters L'_{q}, T'_{q} and T'_{q0} do not exist.

Rotor motion

θ_m angular position of rotor, i.e. angle between one axis attached to the rotor and one attached to the stator. Linked to "electrical" angle θ_r through:

$$\theta_r = p\,\theta_m \qquad p \text{ number of pairs of poles}$$

ω_m mechanical angular speed: $\omega_m = \frac{d}{dt}\theta_m$

ω electrical angular speed: $\omega = \frac{d}{dt}\theta_r = p\omega_m$

Basic equation of rotating masses (friction torque neglected):

$$I\frac{d}{dt}\omega_m = T_m - T_e$$

I moment of inertia of *all* rotating masses

T_m *mechanical* torque provided by prime mover (turbine, diesel motor, etc.)

T_e electromagnetic torque developed by synchronous machine

Motion equation expressed in terms of ω:

$$\frac{I}{p}\frac{d}{dt}\omega = T_m - T_e$$

Dividing by the base torque $T_B = S_B/\omega_{mB}$:

$$\frac{I\omega_{mB}}{pS_B}\frac{d}{dt}\omega = T_{mpu} - T_{epu}$$

Defining the speed in per unit:

$$\omega_{pu} = \frac{\omega}{\omega_N} = \frac{1}{\omega_N}\frac{d}{dt}\theta_r$$

and taking $\omega_{mB} = \omega_B/p = \omega_N/p$, the motion equation becomes:

$$\frac{I\omega_{mB}^2}{S_B}\frac{d}{dt}\omega_{pu} = T_{mpu} - T_{epu}$$

Defining the inertia constant:

$$H = \frac{\frac{1}{2}I\omega_{mB}^2}{S_B}$$

the motion equation is rewritten as:

$$2H\frac{d}{dt}\omega_{pu} = T_{mpu} - T_{epu}$$

Inertia constant H

- called *specific energy*
- ratio $\dfrac{\text{kinetic energy of rotating masses at nominal speed}}{\text{apparent nominal power of machine}}$
- has dimension of a time
- with values in rather narrow interval, whatever the machine power.

H	
thermal plant	hydro plant
$p = 1$: 2 – 4 s $p = 2$: 3 – 7 s	1.5 – 3 s

Relationship between H and launching time t_l

t_l : time to reach the nominal angular speed ω_{mB} when applying to the rotor, initially at rest, the nominal mechanical torque:

$$T_N = \frac{P_N}{\omega_{mB}} = \frac{S_B \cos\phi_N}{\omega_{mB}}$$

P_N: turbine nominal power (in MW) $\cos\phi_N$: nominal power factor

Nominal mechanical torque in per unit: $T_{Npu} = \frac{T_N}{T_B} = \cos\phi_N$

Uniformly accelerated motion: $\omega_{mpu} = \omega_{mpu}(0) + \frac{\cos\phi_N}{2H} t = \frac{\cos\phi_N}{2H} t$

$$\text{At } t = t_l,\ \omega_{mpu} = 1 \qquad \Rightarrow \qquad t_l = \frac{2H}{\cos\phi_N}$$

Remark. Some define t_l with reference to T_B, not T_N. In this case, $t_l = 2H$.

Compensated motion equation

In some simplified models, the damper windings are neglected.
To compensate for the neglected damping torque, a correction term can be added:

$$2H\frac{d}{dt}\omega_{pu} + D(\omega_{pu} - \omega_{sys}) = T_{mpu} - T_{epu} \qquad D \geq 0$$

where ω_{sys} is the system angular frequency (which will be defined in "Power system dynamic modelling under the phasor approximation").

Expression of electromagnetic torque

$$T_e = p(\psi_d i_q - \psi_q i_d)$$

Using the base defined in slide # 16 :

$$\begin{aligned} T_{epu} &= \frac{T_e}{T_B} = \frac{\omega_{mB}}{S_B} p(\psi_d i_q - \psi_q i_d) = \frac{\omega_B}{\sqrt{3}V_B\sqrt{3}I_B}(\psi_d i_q - \psi_q i_d) \\ &= \frac{\psi_d}{\frac{\sqrt{3}V_B}{\omega_B}} \frac{i_q}{\sqrt{3}I_B} - \frac{\psi_q}{\frac{\sqrt{3}V_B}{\omega_B}} \frac{i_d}{\sqrt{3}I_B} = \psi_{dpu} i_{qpu} - \psi_{qpu} i_{dpu} \end{aligned}$$

In per unit, the factor p disappears.

Per unit system for the synchronous machine model

Recall on per unit systems

Consider two magnetically coupled coils with:

$$\psi_1 = L_{11} i_1 + L_{12} i_2 \qquad \psi_2 = L_{21} i_1 + L_{22} i_2$$

For simplicity, we take the same time base in both circuits: $t_{1B} = t_{2B}$

In per unit:
$$\psi_{1pu} = \frac{\psi_1}{\psi_{1B}} = \frac{L_{11}}{L_{1B}} \frac{i_1}{I_{1B}} + \frac{L_{12}}{L_{1B}} \frac{i_2}{I_{1B}} = L_{11pu} i_{1pu} + \frac{L_{12} I_{2B}}{L_{1B} I_{1B}} i_{2pu}$$
$$\psi_{2pu} = \frac{\psi_2}{\psi_{2B}} = \frac{L_{21} I_{1B}}{L_{2B} I_{2B}} i_{1pu} + L_{22pu} i_{2pu}$$

In Henry, one has $L_{12} = L_{21}$. We request to have the same in per unit:

$$L_{12pu} = L_{21pu} \quad \Leftrightarrow \quad \frac{I_{2B}}{L_{1B} I_{1B}} = \frac{I_{1B}}{L_{2B} I_{2B}} \quad \Leftrightarrow \quad S_{1B}\, t_{1B} = S_{2B}\, t_{2B} \quad \Leftrightarrow \quad S_{1B} = S_{2B}$$

A per unit system with $t_{1B} = t_{2B}$ and $S_{1B} = S_{2B}$ is called *reciprocal*

	in the single phase circuit equivalent to stator windings	in each of the d, q windings	in each rotor winding, for instance f
time	$t_B = \frac{1}{\omega_N} = \frac{1}{2\pi f_N}$		
power	$S_B =$ nominal apparent 3-phase		
voltage	V_B: nominal (rms) phase-neutral	$\sqrt{3}V_B$	V_{fB}: to be chosen
current	$I_B = \frac{S_B}{3V_B}$	$\sqrt{3}I_B$	$\frac{S_B}{V_{fB}}$
impedance	$Z_B = \frac{3V_B^2}{S_B}$	$\frac{3V_B^2}{S_B}$	$\frac{V_{fB}^2}{S_B}$
flux	$V_B t_B$	$\sqrt{3}V_B t_B$	$V_{fB} t_B$

The equal-mutual-flux-linkage per unit system

For two magnetically coupled coils, it is shown that (see theory of transformer):

$$\begin{aligned} L_{11} - L_{\ell 1} &= \frac{n_1^2}{\mathcal{R}} \\ L_{12} &= \frac{n_1 n_2}{\mathcal{R}} \\ L_{22} - L_{\ell 2} &= \frac{n_2^2}{\mathcal{R}} \end{aligned}$$

- L_{11} self-inductance of coil 1
- L_{22} self-inductance of coil 1
- $L_{\ell 1}$ leakage inductance of coil 1
- $L_{\ell 2}$ leakage inductance of coil 2
- n_1 number of turns of coil 1
- n_2 number of turns of coil 2
- $\mathcal{R}$ reluctance of the magnetic circuit followed by the magnetic field lines which cross *both* windings; the field is created by i_1 and i_2.

Assume we choose V_{1B} and V_{2B} such that:

$$\frac{V_{1B}}{V_{2B}} = \frac{n_1}{n_2}$$

In order to have the same base power in both circuits:

$$V_{1B} I_{1B} = V_{2B} I_{2B} \qquad \Rightarrow \qquad \frac{I_{1B}}{I_{2B}} = \frac{n_2}{n_1}$$

We have:

$$(L_{11} - L_{\ell 1}) I_{1B} = \frac{n_1^2}{\mathcal{R}} I_{1B} = \frac{n_1^2}{\mathcal{R}} \frac{n_2}{n_1} I_{2B} = \frac{n_1 n_2}{\mathcal{R}} I_{2B} = L_{12} I_{2B} \tag{1}$$

The flux created by I_{2B} in coil 1 is equal to the flux created by I_{1B} in the same coil 1, after removing the part that corresponds to leakages.

Similarly in coil 2:

$$(L_{22} - L_{\ell 2}) I_{2B} = \frac{n_2^2}{\mathcal{R}} I_{2B} = \frac{n_2^2}{\mathcal{R}} \frac{n_1}{n_2} I_{1B} = \frac{n_1 n_2}{\mathcal{R}} I_{1B} = L_{12} I_{1B} \tag{2}$$

This per unit system is said to yield *equal mutual flux linkages* (EMFL)

Alternative definition of base currents

From respectively (1) and (2) :

$$\frac{I_{1B}}{I_{2B}} = \frac{L_{12}}{L_{11} - L_{\ell 1}} \qquad \frac{I_{1B}}{I_{2B}} = \frac{L_{22} - L_{\ell 2}}{L_{12}}$$

A property of this pu system

$$L_{12pu} = \frac{L_{12} I_{2B}}{L_{1B} I_{1B}} = \frac{(L_{11} - L_{\ell 1})}{L_{1B}} = L_{11pu} - L_{\ell 1pu} \tag{3}$$

$$L_{21pu} = \frac{L_{21} I_{1B}}{L_{2B} I_{2B}} = \frac{(L_{22} - L_{\ell 2})}{L_{2B}} = L_{22pu} - L_{\ell 2pu}$$

In this pu system, self-inductance = mutual inductance + leakage reactance.
Does not hold true for inductances in Henry !

The inductance matrix of the two coils takes on the form:

$$\mathbf{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} = \begin{bmatrix} L_{\ell 1} + M & M \\ M & L_{\ell 2} + M \end{bmatrix}$$

Application to synchronous machine

- we have to choose a base voltage (or current) in each rotor winding. Let's first consider the field winding f $(1 \equiv f,\ 2 \equiv d)$
- we would like to use the EMFL per unit system
- we do not know the "number of turns" of the equivalent circuits f, d, etc.
- instead, we can use one of the alternative definitions of base currents:

$$\frac{I_{fB}}{\sqrt{3} I_B} = \frac{L_{dd} - L_\ell}{L_{df}} \quad \Rightarrow \quad I_{fB} = \sqrt{3} I_B \frac{L_{dd} - L_\ell}{L_{df}} \tag{4}$$

  - L_{dd}, L_ℓ can be measured
  - L_{df} can be obtained by measuring the no-load voltage E_q produced by a known field current i_f:

$$E_q = \frac{\omega_N L_{df}}{\sqrt{3}} i_f \quad \Rightarrow \quad L_{df} = \frac{\sqrt{3} E_q}{\omega_N i_f} \tag{5}$$

- the base voltage is obtained from $V_{fB} = \dfrac{S_B}{I_{fB}}$

What about the other rotor windings ?

- one cannot access the d_1, q_1 and q_2 windings to measure L_{dd1}, L_{qq1} et L_{qq2} using formulae similar to (5)
- it can be assumed that there exist base currents I_{d_1B}, I_{q_1B} et I_{q_2B} leading to the EMFL per unit system, but their values are not known
- hence, we cannot compute voltages in Volt or currents in Ampere in those windings (only in pu)
- not a big issue in so far as we do not have to connect anything to those windings (unlike the excitation system to the field winding)...

Numerical example

A machine has the following characteristics:

- nominal frequency: 50 Hz
- nominal apparent power: 1330 MVA
- stator nominal voltage: 24 kV
- $X_d = 0.9\ \Omega \qquad X_\ell = 0.1083\ \Omega$
- field current giving the nominal stator voltage at no-load (and nominal frequency): 2954 A

1. Base power, voltage, impedance, inductance and current at the stator

$$S_B = 1\,330 \text{ MVA}$$

$$U_B = 24\,000 \text{ V} \qquad V_B = \frac{24\,000}{\sqrt{3}} = 13\,856 \text{ V}$$

$$Z_B = \frac{3\ V_B^2}{S_B} = 0.4311\ \Omega \quad \Rightarrow \quad L_B = \frac{Z_B}{\omega_B} = \frac{Z_B}{\omega_N} = \frac{Z_B}{2\,\pi\,50} = 1.378\ 10^{-3} \text{ H}$$

$$I_B = \frac{S_B}{3\ V_B} = \frac{1\,330\ 10^6}{3\ \ 13\,856} = 31\,998 \text{ A}$$

2. Base power, current, voltage, impedance and induct. in field winding

$S_{fB} = S_B = 1\,330$ MVA

$$L_{df} = \frac{\sqrt{3}\ E_q}{\omega_N\ i_f} = \frac{\sqrt{3}\ (24/\sqrt{3})\ 10^3}{2\ \pi\ 50\ \ 2\,954} = 2.586\ 10^{-2}\ \text{H}$$

$$L_{dd} = \frac{X_d}{\omega_N} = \frac{0.9}{2\ \pi\ 50} = 2.865\ 10^{-3}\ \text{H}$$

$$L_\ell = \frac{X_\ell}{\omega_N} = \frac{0.1083}{2\ \pi\ 50} = 3.447\ 10^{-4}\ \text{H}$$

Using Eq. (4) : $$I_{fB} = \sqrt{3}\ I_B\ \frac{L_{dd} - L_\ell}{L_{df}} = 5\,401\ \text{A}$$

$$V_{fB} = \frac{S_{fB}}{I_{fB}} = \frac{1\,330\ 10^6}{5\,401} = 246\,251\ \text{V}$$

A huge value ! This is to be expected since we use the machine nominal power S_B in the field winding, which is not designed to carry such a high power !

$$Z_{fB} = \frac{V_{fB}}{I_{fB}} = \frac{246\,251}{5\,401} = 45.594\ \Omega \quad \Rightarrow \quad L_{fB} = \frac{Z_{fB}}{\omega_B} = \frac{45.594}{2\ \pi\ 50} = 0.14513\ \text{H}$$

3. Convert L_{dd} and L_{df} in per unit

$$L_{dd} = X_d = \frac{0.9}{0.4331} = 2.078 \text{ pu}$$

$$L_\ell = \frac{0.1083}{0.4331} = 0.25 \text{ pu}$$

In per unit, in the EMFL per unit system, Eq. (3) can be used. Hence :

$$L_{df} = L_{dd} - L_\ell = 2.078 - 0.25 = 1.828 \text{ pu} \tag{6}$$

Remarks

- Eq. (6) does *not* hold true in Henry :

  $$L_{dd} - L_\ell = 2.865\ 10^{-3} - 3.447\ 10^{-4} = 2.5203\ 10^{-3} \text{ H} \quad \neq \quad L_{df} = 2.586\ 10^{-2} \text{ H}$$

- in the EMFL per unit system, L_{dd} and L_{df} have comparable values

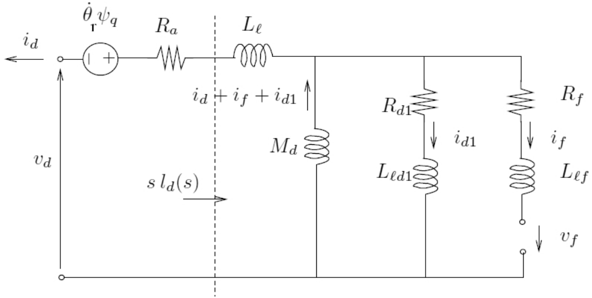
Dynamic equivalent circuits of the synchronous machine

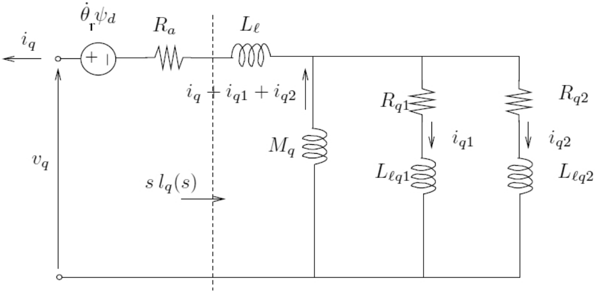
In the EMFL per unit system, the Park inductance matrices take on the simplified form:

$$\mathbf{L}_d = \begin{bmatrix} L_\ell + M_d & M_d & M_d \\ M_d & L_{\ell f} + M_d & M_d \\ M_d & M_d & L_{\ell d1} + M_d \end{bmatrix}$$

$$\mathbf{L}_q = \begin{bmatrix} L_\ell + M_q & M_q & M_q \\ M_q & L_{\ell q_1} + M_q & M_q \\ M_q & M_q & L_{\ell q2} + M_q \end{bmatrix}$$

For symmetry reasons, same leakage inductance L_ℓ in d and q windings


$\dot{\theta}_r \psi_q$
R_a
L_ℓ
i_d
$i_d + i_f + i_{d1}$
R_{d1}
R_f
i_{d1}
i_f
M_d
v_d
$L_{\ell d1}$
$L_{\ell f}$
$s\, l_d(s)$
v_f


$\dot{\theta}_r \psi_d$
R_a
L_ℓ
i_q
$i_q + i_{q1} + i_{q2}$
R_{q1}
R_{q2}
i_{q1}
i_{q2}
M_q
v_q
$L_{\ell q1}$
$L_{\ell q2}$
$s\, l_q(s)$

Modelling of material saturation

Saturation of magnetic material modifies:

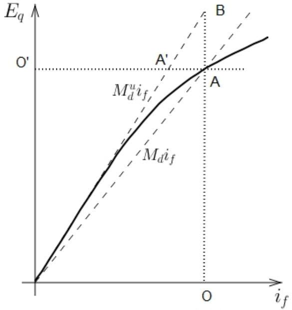
- the machine inductances
- the initial operating point (in particular the rotor position)
- the field current required to obtain a given stator voltage.

Notation

- parameters with the upperscript ^{u} refer to *unsaturated* values
- parameters without this upperscript refer to *saturated* values.

Open-circuit magnetic characteristic

Machine operating at no load, rotating at nominal angular speed ω_N.
Terminal voltage E_q measured for various values of the field current i_f.



saturation factor : $k = \frac{OA}{OB} = \frac{O'A'}{O'A} < 1$

a standard model : $k = \frac{1}{1 + m(E_q)^n} \quad m, n > 0$

characteristic in d axis (field due to i_f only)

In per unit:
$$E_{qpu} = \frac{\omega_N L_{df} i_f}{\sqrt{3}\, V_B} = \frac{\omega_N L_{df} I_{fB}}{\sqrt{3}\, V_B} i_{fpu} = \frac{\omega_N L_{df}}{\sqrt{3}\, V_B} \sqrt{3} I_B \frac{L_{dd} - L_\ell}{L_{df}} i_{fpu}$$
$$= \frac{\omega_N}{Z_B}(L_{dd} - L_\ell) i_{fpu} = M_{dpu} i_{fpu}$$

Dropping the $_{pu}$ notation and introducing k:

$$E_q = M_d i_f = k\, M_d^u i_f$$

Leakage and air gap flux

The flux linkage in the d winding is decomposed into:

$$\psi_d = L_\ell i_d + \psi_{ad}$$

$L_\ell i_d$: *leakage flux*, not subject to saturation (path mainly in the air)
ψ_{ad}: direct-axis component of the *air gap flux*, subject to saturation.

Expression of ψ_{ad}:

$$\psi_{ad} = \psi_d - L_\ell i_d = M_d(i_d + i_f + i_{d1})$$

Expression of ψ_{aq}:

$$\psi_{aq} = \psi_q - L_\ell i_q = M_q(i_q + i_{q1} + i_{q2})$$

Considering that the d and q axes are orthogonal, the *air gap* flux is given by:

$$\psi_{ag} = \sqrt{\psi_{ad}^2 + \psi_{aq}^2} \qquad (7)$$

Saturation characteristic in loaded conditions

- Saturation is different in the d and q axes, especially for a salient pole machine (air gap larger in q axis !). Hence, different saturation factors (say, k_d and k_q) should be considered
- in practice, however, it is quite common to have only the direct-axis saturation characteristic
- in this case, the characteristic is used along any axis (not just d) as follows
- in no-load conditions, we have

$$\psi_{ad} = M_d i_f \quad \text{and} \quad \psi_{aq} = 0 \quad \Rightarrow \quad \psi_{ag} = M_d i_f$$

$$M_d = k M_d^u = \frac{M_d^u}{1 + m(E_q)^n} = \frac{M_d^u}{1 + m(M_d i_f)^n} = \frac{M_d^u}{1 + m(\psi_{ag})^n}$$

- it is assumed that this relation still holds true with the combined air gap flux ψ_{ag} given by (7).

Summary: complete model (variables in per unit)

$$\psi_d = L_\ell i_d + \psi_{ad} \qquad \psi_q = L_\ell i_q + \psi_{aq}$$

$$\psi_f = L_{\ell f} i_f + \psi_{ad} \qquad \psi_{q1} = L_{\ell q1} i_{q1} + \psi_{aq}$$

$$\psi_{d1} = L_{\ell d1} i_{d1} + \psi_{ad} \qquad \psi_{q2} = L_{\ell q2} i_{q2} + \psi_{aq}$$

$$\psi_{ad} = M_d(i_d + i_f + i_{d1}) \qquad \psi_{aq} = M_q(i_q + i_{q1} + i_{q2})$$

$$M_d = \frac{M_d^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n} \qquad M_q = \frac{M_q^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n}$$

$$v_d = -R_a i_d - \omega\psi_q - \frac{d\psi_d}{dt} \qquad v_q = -R_a i_q + \omega\psi_d - \frac{d\psi_q}{dt}$$

$$\frac{d}{dt}\psi_f = v_f - R_f i_f \qquad \frac{d}{dt}\psi_{q1} = -R_{q1} i_{q1}$$

$$\frac{d}{dt}\psi_{d1} = -R_{d1} i_{d1} \qquad \frac{d}{dt}\psi_{q2} = -R_{q2} i_{q2}$$

$$2H\frac{d}{dt}\omega = T_m - (\psi_d i_q - \psi_q i_d) \qquad \frac{1}{\omega_N}\frac{d}{dt}\theta_r = \omega$$

Model simplifications

Constant rotor speed approximation $\dot{\theta}_r \simeq \omega_N$ $(\omega = 1 \text{ pu})$

Examples showing that $\dot{\theta}_r$ does not depart much from the nominal value ω_N :

1. oscillation of θ_r with a magnitude of 90^o and period of 1 second superposed to the uniform motion at synchronous speed:

$$\theta_r = \theta_r^o + 2\pi f_N t + \frac{\pi}{2} \sin 2\pi t \quad \Rightarrow \quad \dot{\theta}_r = 100\pi + \pi^2 \cos 2\pi t \simeq 314 + 10 \cos 2\pi t$$

   at its maximum, it deviates from nominal by $10/314 = 3$ % only.

2. in a large interconnected system, after primary frequency control, frequency settles at $f \neq f_N$. $|f - f_N| = 0.1$ Hz is already a large deviation. In this case, machine speeds deviate from nominal by $0.1/50 = 0.2$ % only.
3. a small isolated system may experience larger frequency deviations. But even for $|f - f_N| = 1$ Hz, the machine speeds deviate from nominal by $1/50 = 2$ % only.

The phasor (or quasi-sinusoidal) approximation

- Underlies a large class of power system dynamic simulators
- considered in detail in the following lectures
- for the synchronous machine, it consists of neglecting the "transformer voltages" $\frac{d\psi_d}{dt}$ and $\frac{d\psi_d}{dt}$ in the stator Park equations
- this leads to neglecting some fast varying components of the network response, and allows the voltage and currents to be treated as a sinusoidal with time-varying amplitudes and phase angles (hence the name)
- at the same time, three-phase balance is also assumed.

Thus, the stator Park equations become (in per unit, with $\omega = 1$):

$$\begin{aligned} v_d &= -R_a i_d - \psi_q \\ v_q &= -R_a i_q + \psi_d \end{aligned}$$

and ψ_d and ψ_q are now algebraic, instead of differential, variables.

Hence, they may undergo a discontinuity after of a network disturbance.

The "classical" model of the synchronous machine

Very simplified model used :

- in some analytical developments
- in qualitative reasoning dealing with transient (angle) stability
- for fast assessment of transient (angle) stability.

"Classical" refers to a model used when there was little computational power.

Approximation # 0. We consider the phasor approximation.

Approximation # 1. The damper windings d_1 et q_2 are ignored.

- The damping of rotor oscillations is going to be underestimated.

Approximation # 2. The stator resistance R_a is neglected.

- This is very acceptable.

The stator Park equations become :

$$\begin{aligned} v_d &= -\psi_q = -L_{qq} i_q - L_{qq_1} i_{q_1} \\ v_q &= \psi_d = L_{dd} i_d + L_{df} i_f \end{aligned}$$

Expressing i_f (resp. i_{q1}) as function of ψ_f and i_d (resp. ψ_{q1} and i_q) :

$$\psi_f = L_{ff}\, i_f + L_{df}\, i_d \quad \Rightarrow \quad i_f = \frac{\psi_f - L_{df}\, i_d}{L_{ff}}$$

$$\psi_{q1} = L_{q1q1}\, i_{q1} + L_{qq1}\, i_{q1} \quad \Rightarrow \quad i_{q1} = \frac{\psi_{q1} - L_{qq1}\, i_q}{L_{q1q1}}$$

and introducing into the stator Park equations :

$$v_d = -\underbrace{(L_{qq} - \frac{L_{qq1}^2}{L_{q1q1}})}_{L'_q}\, i_q \underbrace{- \frac{L_{qq1}}{L_{q1q1}} \psi_{q1}}_{e'_d} = -X'_q\, i_q + e'_d \qquad (8)$$

$$v_q = \underbrace{(L_{dd} - \frac{L_{df}^2}{L_{ff}})}_{L'_d}\, i_d + \underbrace{\frac{L_{df}}{L_{ff}} \psi_f}_{e'_q} = X'_d\, i_d + e'_q \qquad (9)$$

e'_d and e'_q :

- are called the *e.m.f. behind transient reactances*
- are proportional to magnetic fluxes; hence, they cannot vary much after a disturbance, unlike the rotor currents i_f and i_{q1}.

Approximation # 3. The e.m.f. e'_d and e'_q are assumed constant.

- This is valid over no more than - say - one second after a disturbance;
- over this interval, a single rotor oscillation can take place; hence, damping cannot show its effect (i.e. Approximation # 1 is not a concern).

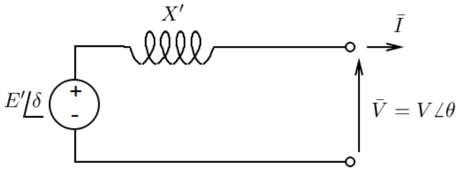
Equations (8, 9) are similar to the Park equations in steady state, except for the presence of an e.m.f. in the d axis, and the replacement of the synchronous by the transient reactances.

Approximation # 4. The transient reactance is the same in both axes : $X'_d = X'_q$.

- Questionable, but experiences shows that X'_q has less impact ...

If $X'_d = X'_q$, Eqs. (8, 9) can be combined in a single phasor equation, with the corresponding equivalent circuit:

$$\bar{V} + jX'_d\,\bar{I} = \bar{E}' = E'\angle\delta$$



Rotor motion. This is the only dynamics left !

e'_d and e'_q are constant. Hence, $\bar{E}'$ is fixed with respect to d and q axes, and δ differs from θ_r by a constant.

Therefore, $\frac{1}{\omega_N}\frac{d}{dt}\theta_r = \omega$ can be rewritten as : $\frac{1}{\omega_N}\frac{d}{dt}\delta = \omega$

The rotor motion equation:

$$2H\,\frac{d}{dt}\omega = T_m - T_e$$

is transformed to involve powers instead of torques. Multiplying by ω:

$$2H\,\omega\,\frac{d}{dt}\omega = \omega T_m - \omega T_e$$

$\omega T_m =$ mechanical power P_m of the turbine

$\omega T_e = p_{r\rightarrow s} = p_T(t) + p_{Js} + \frac{dW_{ms}}{dt} \simeq P$ (active power produced)

since we assume three-phase balanced AC operation, and R_a is neglected

Approximation # 5. We assume $\omega \simeq 1$ and replace $2H\omega$ by $2H$

- very acceptable, already justified.

Thus we have:

$$2H \frac{d}{dt}\omega = P_m - P$$

where P can be derived from the equivalent circuit:

$$P = \frac{E'V}{X'} \sin(\delta - \theta)$$