ELEC0047 - Power system dynamics, control and stability

Dynamics of the induction machine

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Brief recall

Induction or *asynchronous* machine

- motor widely used in industry, tertiary sector, etc.
- sometimes also as small generator

Principle of operation

Stator:

- three-phrase windings carrying three-phase currents of angular frequency ω_s
- produces a magnetic field rotating at angular speed ω_s
- a single pair of poles is assumed for simplicity.

Rotor:

- rotates at a speed $\omega_r \neq \omega_s$ characterized by the motor *slip* :

$$s = \frac{\omega_s - \omega_r}{\omega_s}$$

- can be modeled with a set of three-phase windings
- currents induced in these windings have angular frequency $\omega_s - \omega_r = s\omega_s$
- and produce a magnetic field rotating at angular speed $s\omega_s$ with respect to the rotor, i.e. $s\omega_s + \omega_r = \omega_s$ with respect to the stator.

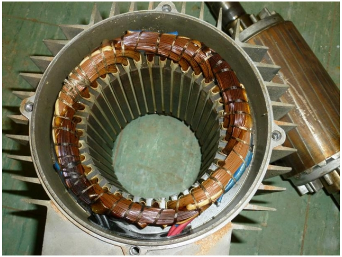
Both rotating magnetic fields are fixed with respect to each other.

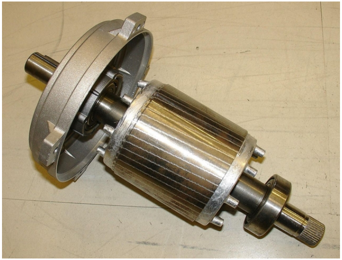
Their interaction creates the electromagnetic torque.

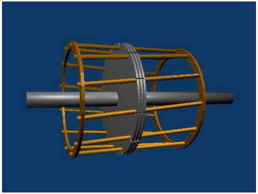
Two types of machines: squirrel-cage and wound rotors

Squirrel-cage rotor

- non insulated aluminum or copper bars inserted in slots, connected at their ends to allow the currents to flow
- simple construction, easy maintenance, reliable operation
- possible presence of a second cage aimed at providing a larger starting torque (non considered here).

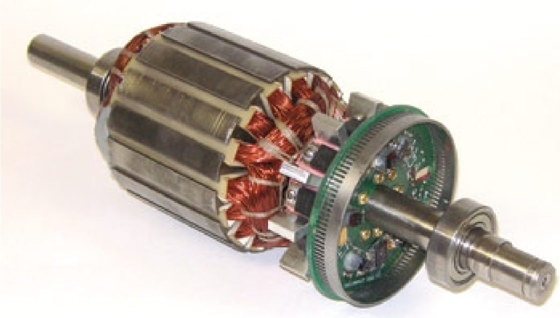




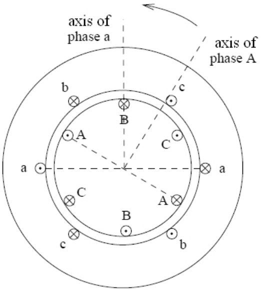


Wound rotor

- the rotor carries insulated three-phase windings, which are accessed through sliprings and brushes
- used when the rotor circuits have to be accessed, e.g. to control
    - the starting torque (external resistance)
    - the starting current
    - the rotor speed
- construction and maintenance are more expensive.



Modelling the induction machine



Motor sign convention at both stator and rotor.

$$v_a = R_s i_a + \frac{d\psi_a}{dt} \qquad v_b = R_s i_b + \frac{d\psi_b}{dt} \qquad v_c = R_s i_c + \frac{d\psi_c}{dt}$$

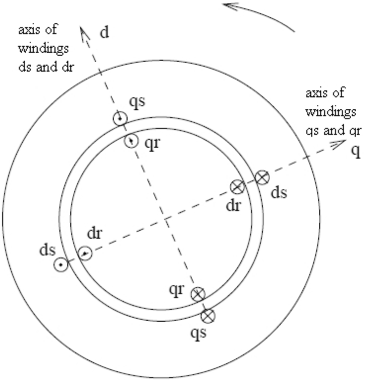
$$0 = R_r i_A + \frac{d\psi_A}{dt} \qquad 0 = R_r i_B + \frac{d\psi_B}{dt} \qquad 0 = R_r i_C + \frac{d\psi_C}{dt}$$

R_s : resistance of one stator circuit $\qquad$ R_r : resistance of one rotor circuit

single pair of poles assumed for simplicity of notation

Park transformation, equations and inductance matrix

- Several reference frames can be used, depending on the application
- we use d and q reference axes which rotate at the angular speed ω_s
- *both* stator and rotor windings are transformed into this reference frame
- this yields new, equivalent windings which are all fixed wrt each other.



ω_r: rotor speed

	original windings	transformed into	relative speed
stator	a, b, c	ds, qs, os	ω_s
rotor	A, B, C	dr, qr, or	$\omega_s - \omega_r$

By similarity with the derivations of the synchronous machine :

$$
\begin{aligned}
v_{ds} &= R_s i_{ds} + \omega_s \psi_{qs} + \frac{d\psi_{ds}}{dt} \\
v_{qs} &= R_s i_{qs} - \omega_s \psi_{ds} + \frac{d\psi_{qs}}{dt} \\
v_{os} &= R_s i_{os} + \frac{d\psi_{os}}{dt} \\
0 &= R_r i_{dr} + (\omega_s - \omega_r)\psi_{qr} + \frac{d\psi_{dr}}{dt} \\
0 &= R_r i_{qr} - (\omega_s - \omega_r)\psi_{dr} + \frac{d\psi_{qr}}{dt} \\
0 &= R_r i_{or} + \frac{d\psi_{or}}{dt}
\end{aligned}
$$

$$
\begin{bmatrix} \psi_{ds} \\ \psi_{qs} \\ \psi_{os} \\ \psi_{dr} \\ \psi_{qr} \\ \psi_{or} \end{bmatrix}
=
\begin{bmatrix}
L_{ss} & & & L_{sr} & & \\
& L_{ss} & & & L_{sr} & \\
& & L_{os} & & & \\
L_{sr} & & & L_{rr} & & \\
& L_{sr} & & & L_{rr} & \\
& & & & & L_{or}
\end{bmatrix}
\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{os} \\ i_{dr} \\ i_{qr} \\ i_{or} \end{bmatrix}
$$

Typical values

R_s	0.01 - 0.12 pu	R_r	0.01 - 0.13 pu
$L_{ss} - L_{sr}$	0.07 - 0.15 pu	$L_{rr} - L_{sr}$	0.06 - 0.18 pu
L_{sr}	1.8 - 3.8 pu		

per unit values on the machine base

Energy, power and torque

Stator power balance

Instantaneous power entering the stator =

Joule losses in stator p_{Js}
+ d/dt magnetic energy in stator windings W_{ms}
+ power passing from stator to rotor $p_{s\rightarrow r}$ (what type of power is it ?)

$$\begin{aligned} p_T(t) &= v_a i_a + v_b i_b + v_c i_c = v_{ds} i_{ds} + v_{qs} i_{qs} + v_{os} i_{os} \\ &= (R_s i_{ds}^2 + R_s i_{qs}^2 + R_s i_{os}^2) + (i_{ds} \frac{d\psi_{ds}}{dt} + i_{qs} \frac{d\psi_{qs}}{dt} + i_{os} \frac{d\psi_{os}}{dt}) \\ &\quad + \omega_s (\psi_{qs} i_{ds} - \psi_{ds} i_{qs}) \end{aligned}$$

Hence:

$$p_{s\rightarrow r} = \omega_s (\psi_{qs} i_{ds} - \psi_{ds} i_{qs}) \qquad (1)$$

Rotor power balance

Power passing from stator to rotor $p_{s\rightarrow r} =$

Joule losses in rotor p_{Jr} + d/dt magnetic energy in rotor windings W_{mr}
+ d/dt kinetic energy W_c + power transferred to the mechanical load P_m.

From the Park equations :

$$v_{dr} i_{dr} + v_{qr} i_{qr} + v_{or} i_{or} = 0$$

$$(R_r i_{dr}^2 + R_r i_{qr}^2 + R_r i_{or}^2) + (i_{dr}\frac{d\psi_{dr}}{dt} + i_{qr}\frac{d\psi_{qr}}{dt} + i_{or}\frac{d\psi_{or}}{dt}) + (\omega_s - \omega_r)(\psi_{qr} i_{dr} - \psi_{dr} i_{qr}) = 0$$

$$p_{Jr} + \frac{dW_{mr}}{dt} = -(\omega_s - \omega_r)(\psi_{qr} i_{dr} - \psi_{dr} i_{qr})$$

Hence, the above rotor power balance equation can be rewritten as :

$$p_{s\rightarrow r} = -(\omega_s - \omega_r)(\psi_{qr} i_{dr} - \psi_{dr} i_{qr}) + \frac{dW_c}{dt} + P_m$$

Replacing $p_{s\rightarrow r}$ by (1) and using the rotor motion equation :

$$(\omega_s - \omega_r)(\psi_{qr} i_{dr} - \psi_{dr} i_{qr}) + \omega_s(\psi_{qs} i_{ds} - \psi_{ds} i_{qs}) = \omega_r T_e$$

Expressions of torque

$$\psi_{qs} i_{ds} - \psi_{ds} i_{qs} = (L_{ss} i_{qs} + L_{sr} i_{qr}) i_{ds} - (L_{ss} i_{ds} + L_{sr} i_{dr}) i_{qs} = L_{sr}(i_{qr} i_{ds} - i_{dr} i_{qs})$$

$$\begin{aligned} \psi_{qr} i_{dr} - \psi_{dr} i_{qr} &= (L_{rr} i_{qr} + L_{sr} i_{qs}) i_{dr} - (L_{rr} i_{dr} + L_{sr} i_{ds}) i_{qr} = L_{sr}(i_{qs} i_{dr} - i_{ds} i_{qr}) \\ &= -(\psi_{qs} i_{ds} - \psi_{ds} i_{qs}) \end{aligned}$$

Hence :

$$T_e = \psi_{qs} i_{ds} - \psi_{ds} i_{qs} = \psi_{dr} i_{qr} - \psi_{qr} i_{dr} = L_{sr}(i_{qr} i_{ds} - i_{dr} i_{qs}).$$

Remarks

- The above derivation shows that $p_{s \to r} = \omega_s T_e$
- $p_{s \to r}$ is both of electromagnetic and mechanical nature
- the expression of T_e looks very similar to that of the synchronous machine
- but both machines behave quite differently
- in particular, in the synchronous machine, $p_{s \to r}$ is of mechanical nature only.

Rotor motion equation

Following the same derivation as for the synchronous machine yields :

$$2H\frac{d}{dt}\omega_r = T_e - T_m$$

where H is the inertia constant, in second
ω_r, T_e and T_m are in per unit
t is in second.

Mechanical torque T_m : varies with the rotor speed ω_r

A common model is :

$$T_m = T_{mo}\left(A\omega_r^2 + B\omega_r + C\right) \qquad \text{with} \quad A + B + C = 1$$

where :

T_{mo} is the torque value at synchronous speed, i.e. when $\omega_r = 1$

A, B et C depend on the driven mechanical load.

Typical inertia and torque parameters

component	A	B	C	H (s)
heat pump, air conditioning	0.2	0.0	0.8	0.28
refrigerator, freezer	0.2	0.0	0.8	0.28
dishwasher	1.0	0.0	0.0	0.28
clothes washer	1.0	0.0	0.0	1.50
clothes dryer	1.0	0.0	0.0	1.30
pumps, fans, other motors	1.0	0.0	0.0	0.70
small industrial motor	1.0	0.0	0.0	0.70
large industrial motor	1.0	0.0	0.0	1.50
power plant auxiliaries	1.0	0.0	0.0	1.50
agricultural water pump	1.0	0.0	0.0	0.4

Model under the phasor approximation

Neglecting transformer voltages and dropping the "*os*" winding:

$$v_{ds} = R_s i_{ds} + \omega_s \psi_{qs} \quad (2)$$

$$v_{qs} = R_s i_{qs} - \omega_s \psi_{ds} \quad (3)$$

The other equations are unchanged. Dropping the "*or*" winding:

$$\frac{d\psi_{dr}}{dt} = -R_r i_{dr} - (\omega_s - \omega_r)\psi_{qr} \quad (4)$$

$$\frac{d\psi_{qr}}{dt} = -R_r i_{qr} + (\omega_s - \omega_r)\psi_{dr} \quad (5)$$

$$\psi_{ds} = L_{ss} i_{ds} + L_{sr} i_{dr} \quad (6)$$

$$\psi_{qs} = L_{ss} i_{qs} + L_{sr} i_{qr} \quad (7)$$

$$\psi_{dr} = L_{sr} i_{ds} + L_{rr} i_{dr} \quad (8)$$

$$\psi_{qr} = L_{sr} i_{qs} + L_{rr} i_{qr} \quad (9)$$

$$2H \frac{d}{dt}\omega_r = \psi_{dr} i_{qr} - \psi_{qr} i_{dr} - T_{mo}\left(A\omega_r^2 + B\omega_r + C\right) \quad (10)$$

Third-order model of the (single-cage) induction machine.

Simplified (first-order) model

- Rotor windings contribute with fast transients
- approximation: assume their dynamics infinitely fast, and set $d\psi_r/dt = \mathbf{0}$
- this yields a first-order model, with rotor motion as the only dynamics.

At the rotor:

$$
\begin{aligned}
0 &= R_r i_{dr} + (\omega_s - \omega_r)\psi_{qr} = R_r i_{dr} + (\omega_s - \omega_r)L_{rr} i_{qr} + (\omega_s - \omega_r)L_{sr} i_{qs} \\
0 &= R_r i_{qr} - (\omega_s - \omega_r)\psi_{dr} = R_r i_{dr} - (\omega_s - \omega_r)L_{rr} i_{dr} - (\omega_s - \omega_r)L_{sr} i_{ds}
\end{aligned}
$$

Dividing by $\dfrac{\omega_s - \omega_r}{\omega_s}$:

$$
0 = \frac{\omega_s R_r}{\omega_s - \omega_r} i_{dr} + \omega_s L_{rr} i_{qr} + \omega_s L_{sr} i_{qs} \quad (11)
$$

$$
0 = \frac{\omega_s R_r}{\omega_s - \omega_r} i_{qr} - \omega_s L_{rr} i_{dr} - \omega_s L_{sr} i_{ds} \quad (12)
$$

$\dfrac{\omega_s - \omega_r}{\omega_s}$ is the *rotor slip with respect to* ω_s.

At the stator:

$$\begin{aligned} v_{ds} &= R_s i_{ds} + \omega_s (L_{ss} i_{qs} + L_{sr} i_{qr}) && (13) \\ v_{qs} &= R_s i_{qs} - \omega_s (L_{ss} i_{ds} + L_{sr} i_{dr}) && (14) \end{aligned}$$

By analogy with the synchronous machine, one can interpret :

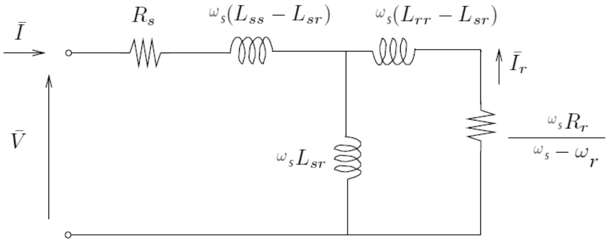
- i_{ds} and i_{qs} as projections on the (d, q) axes of a rotating vector representing the current in phase a, with corresponding phasor $\bar{I}$;
- i_{dr} and i_{qr} as projections on (d, q) axes of a rotating vector representing the current in one rotor winding, seen from stator, with corresponding phasor $\bar{I}_r$.

Eqs. (11, 12) and (13, 14) can be combined into complex equations:

$$\overline{V} = R_s \bar{I} + j\omega_s L_{ss} \bar{I} + j\omega_s L_{sr} \bar{I}_r$$

$$0 = \frac{\omega_s \, R_r}{\omega_s - \omega_r} \bar{I}_r + j\omega_s L_{rr} \bar{I}_r + j\omega_s L_{sr} \bar{I}$$

This corresponds to the equivalent circuit :

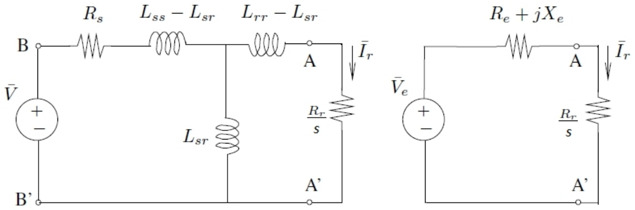


in which :

- the "electrical part" is static
- ω_r varies according to the rotor motion equation (10).

Steady-state torque-slip characteristic

Motor powered under a stator voltage $\bar{V}$



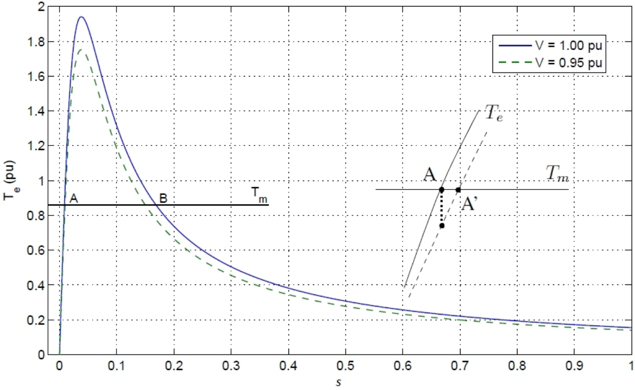
$$\bar{V}_e = \bar{V}\frac{j\omega_s L_{sr}}{R_s + j\omega_s L_{ss}}$$

$$R_e + jX_e = j\omega_s(L_{rr} - L_{sr}) + \frac{j\omega_s L_{sr}(R_s + j\omega_s(L_{ss} - L_{sr}))}{R_s + j\omega_s L_{ss}} = j\omega_s L_{rr} + \frac{\omega_s L_{sr}^2}{R_s + j\omega_s L_{ss}}$$

$$p_{s\to r} = \frac{R_r}{s} I_r^2 = \omega_s T_e \quad \Rightarrow \quad T_e = \frac{1}{\omega_s}\frac{R_r}{s} I_r^2 = \frac{1}{\omega_s}\frac{R_r}{s}\frac{V_e^2}{(R_e + \frac{R_r}{s})^2 + X_e^2}$$

Example Large industrial motor :

$$L_{ss} = 3.867, L_{sr} = 3.800, L_{rr} = 3.970, R_s = 0.013, R_r = 0.009 \text{ pu}$$



Equilibrium points correspond to: $T_e = T_m$ A : stable B : unstable

Maximum torque T_e^{max} proportional to V_e^2, and hence to V^2.

Motor response to a step decrease of voltage V

T_m assumed constant (for simplicity; valid for small speed variations)

- very first instants: inertia of rotating masses $\Rightarrow$ motor slip unchanged $\Rightarrow$ R_r/s unchanged $\Rightarrow$ motor behaves as a constant admittance
- soon after: $T_e < T_m$ $\Rightarrow$ the motor decelerates $\Rightarrow$ moves to equilibrium A'
- at the new operating point :

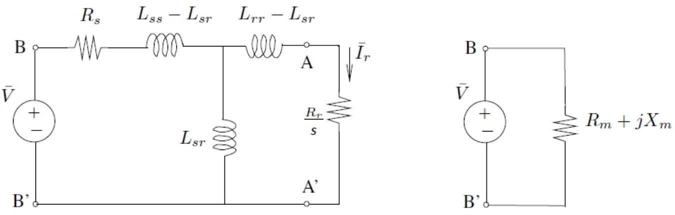
$$p_{s\rightarrow r} = \omega_s T_e = \omega_s T_m$$

Conclusion:

- the induction motor is a load which, after a voltage disturbance, restores an internally consumed active power ($p_{s\rightarrow r}$) to its pre-disturbance value
- it does so rather fast: new equilibrium reached in less than 1 s typically
- from system operator viewpoint: decreasing the network voltage does not relieve the system in terms of load active power :-(

After a large enough voltage drop, $T_e^{max} < T_m$: the motor *stalls* $\Rightarrow$ s increases $\Rightarrow$ I increases a lot $\Rightarrow$ the motor is eventually tripped by its thermal protection

Variations of motor active and reactive powers with voltage and frequency



$$P = \frac{R_m}{R_m^2 + X_m^2} V^2 \qquad Q = \frac{X_m}{R_m^2 + X_m^2} V^2$$

$$\begin{aligned} R_m + jX_m &= R_s + j\omega_s (L_{ss} - L_{sr}) + \frac{j\omega_s L_{sr} (\frac{R_r}{s} + j\omega_s (L_{rr} - L_{sr}))}{\frac{R_r}{s} + j\omega_s L_{rr}} \\ &= R_s + j\omega_s L_{ss} + \frac{\omega_s^2 L_{sr}^2}{\frac{R_r}{s} + j\omega_s L_{rr}} \end{aligned}$$

The motor slip s is given by the torque equilibrium condition:

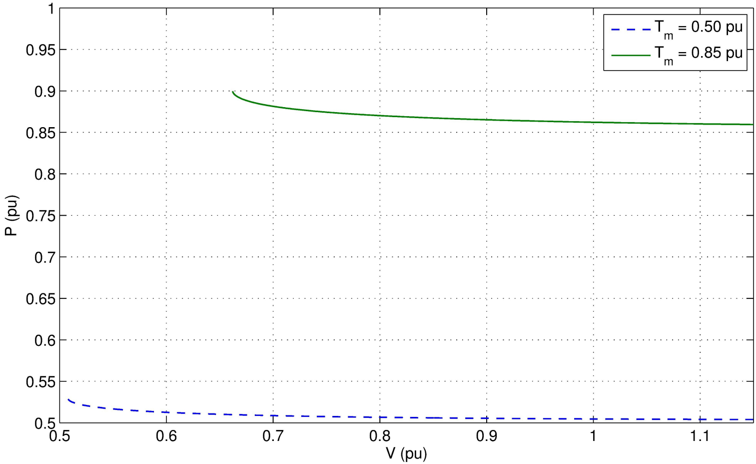
$$T_m = T_e \quad \Leftrightarrow \quad T_m = \frac{1}{\omega_s} \frac{R_r}{s} \frac{V_e^2}{(R_e + \frac{R_r}{s})^2 + X_e^2} \tag{15}$$

<u>Procedure</u>.

For a given set of (V, ω_s, T_m) values :

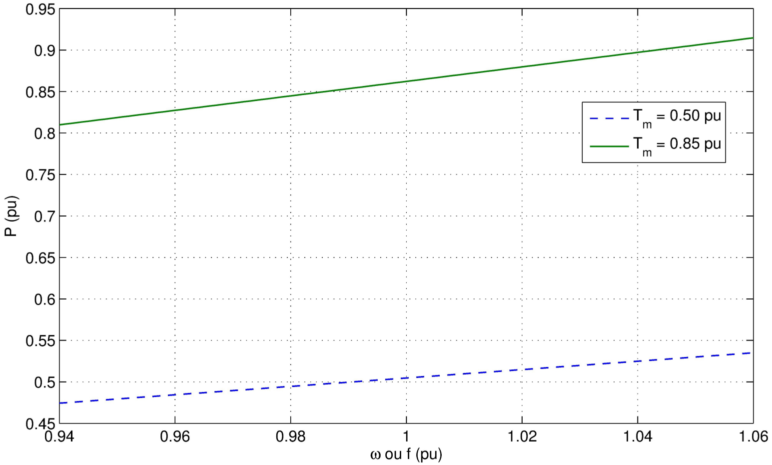
1. compute V_e, R_e and X_e (see slide # 19)
2. solve (15) to obtain s
   - solve the equation with respect to $\frac{R_r}{s}$, treated as intermediate variable
   - from which s is easily obtained.
3. compute R_m and X_m (see slide # 22)
4. compute P and Q (see slide # 22).

Variation of active power P with voltage V



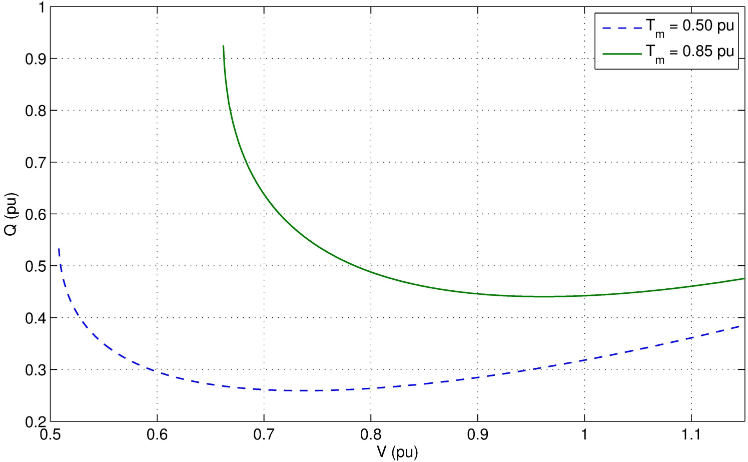
Exercise: show that, if R_s is neglected, P is constant (down to the stalling point)

Variation of active power P with angular frequency ω_s (or frequency f)



Exercise: show that, if R_s is neglected, P varies linearly with f

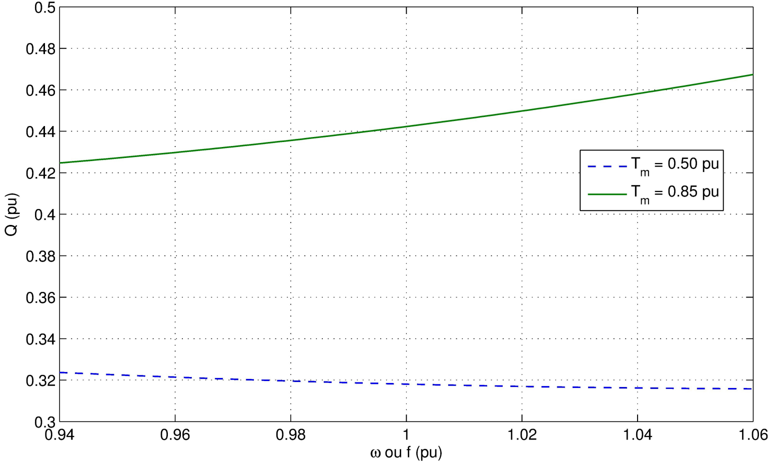
Variation of reactive power Q with voltage V



at high V values: power consumed in L_{sr} dominates; it varies quadratically with V

at low V values: power consumed in $L_{ss} - L_{rr}$ and $L_{rr} - L_{sr}$ dominates

Variation of reactive power Q with angular frequency ω_s (or frequency f)



The slope is positive or negative, depending upon the mechanical load !