ELEC0047 - Power system dynamics, control and stability

Behaviour of synchronous machine during a short-circuit (a simple example of electromagnetic transients)

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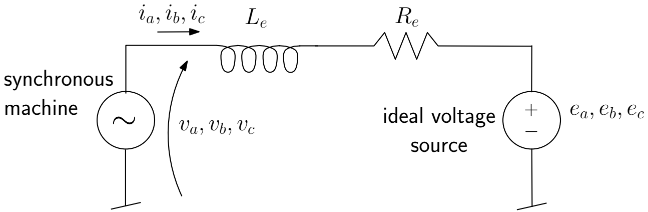
Objectives of this lecture

- Recall the Park model of synchronous machines
- give an example of electromagnetic transient simulation
- prepare the derivation of the corresponding model under the "phasor approximation'

System modelling



Network represented by a simple Thévenin equivalent :

- resistance R_e and inductance L_e in each phase
- no magnetic coupling between phases, for simplicity

Machine :

- only the field winding f in the d axis
- only one damper winding $q1$ in the q axis
- rotor speed $\dot{\theta}_r$ assumed constant
  - focus on short-lasting electromagnetic transients: speed has no time to change
- constant excitation voltage V_f
  - it is assumed that the automatic voltage regulator has no time to react

Network equations

$$
\begin{aligned}
v_a - e_a &= R_e i_a + L_e \frac{di_a}{dt} \qquad \text{with} \qquad e_a = \sqrt{2}E\cos(\omega_N t + \theta_e) && (1)\\
v_b - e_b &= R_e i_b + L_e \frac{di_b}{dt} \qquad \text{with} \qquad e_b = \sqrt{2}E\cos(\omega_N t + \theta_e - \frac{2\pi}{3}) && (2)\\
v_c - e_c &= R_e i_c + L_e \frac{di_c}{dt} \qquad \text{with} \qquad e_b = \sqrt{2}E\cos(\omega_N t + \theta_e - \frac{4\pi}{3}) && (3)
\end{aligned}
$$

Park transformation

$$
\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = \mathcal{P} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \mathcal{P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (4)
$$

$$
\text{with } \mathcal{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{4\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{4\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad (5)
$$

where $\theta_r = \theta_r^o + \omega_N t$

Machine equations

$$\psi_d = L_{dd} i_d + L_{df} i_f \quad (6)$$
$$\psi_q = L_{qq} i_q + L_{qq1} i_{q1} \quad (7)$$
$$\psi_o = L_{oo} i_o \quad (8)$$
$$\psi_f = L_{ff} i_f + L_{df} i_d \quad (9)$$
$$\psi_{q1} = L_{q1q1} i_{q1} + L_{qq1} i_q \quad (10)$$
$$v_d = -R_a i_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt} \quad (11)$$
$$v_q = -R_a i_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt} \quad (12)$$
$$v_o = -R_a i_o - \frac{d\psi_o}{dt} \quad (13)$$
$$V_f = R_f i_f + \frac{d\psi_f}{dt} \quad (14)$$
$$0 = R_{q1} i_{q1} + \frac{d\psi_{q1}}{dt} \quad (15)$$

Variable - equations balance

- 19 variables : $v_a, v_b, v_c, i_a, i_b, i_c, v_d, v_q, v_o, i_d, i_q, i_o, \psi_d, \psi_q, \psi_o, \psi_f, \psi_{q1}, i_f, i_{q1}$
- 19 equations : (1 - 3), 6 eqs. in (4), (6 - 15)

Remarks

- The model is made up of Differential-Algebraic Equations (DAEs)
- some of the variables and some of the equations could be eliminated but the additional computational effort of keeping all of them is negligible[1]
- θ_r being known, the equations are linear with respect to the unknowns
- some coefficients in these equations vary with time.

[1] not to mention the risk of introducing mistakes in analytical manipulations !

Passing the equations in per unit

In each phase of the stator (a, b, c):

- base voltage V_B = nominal RMS phase-to-neutral voltage (kV)
- base power S_B = three-phase apparent power (MVA)
- base current $I_B = S_B/3V_B$, base flux $\psi_B = V_B/\omega_N$, etc.

In each park winding (d, q, o):

- base voltage $V_{PB} = \sqrt{3}V_B$
- base power $= S_B$
- base current $I_{PB} = S_B/V_{PB} = \sqrt{3}I_B$, base flux $\psi_B = V_{PB}/\omega_N$, etc.

In the rotor windings f and $q1$:

- we use the reciprocal "Equal Mutual Flux Linkages" per unit system
- the latter is such that $L_{dd} = L_{df} + L_\ell$ and $L_{qq} = L_{qq1} + L_\ell$ in per unit, where L_ℓ is the leakage inductance (same for both d and q windings)

After passing in per unit:

- $\dot{\theta}_r = 1$ pu in Eqs. (11, 12)
- the equations are unchanged, except that each time derivative is multiplied by $1/\omega_N$, since we keep the time in second (not in pu)

Equations converted in per unit and rearranged

$$\frac{1}{\omega_N}\frac{di_a}{dt} = -\frac{R_e}{L_e}i_a + \frac{1}{L_e}v_a - \frac{1}{L_e}e_a \quad (16)$$

$$\frac{1}{\omega_N}\frac{di_b}{dt} = -\frac{R_e}{L_e}i_b + \frac{1}{L_e}v_b - \frac{1}{L_e}e_b \quad (17)$$

$$\frac{1}{\omega_N}\frac{di_c}{dt} = -\frac{R_e}{L_e}i_c + \frac{1}{L_e}v_c - \frac{1}{L_e}e_c \quad (18)$$

$$0 = \frac{\sqrt{2}}{3}\left[\cos(\theta_r)v_a + \cos(\theta_r - \frac{2\pi}{3})v_b + \cos(\theta - \frac{4\pi}{3})v_c\right] - v_d \quad (19)$$

$$0 = \frac{\sqrt{2}}{3}\left[\sin(\theta_r)v_a + \sin(\theta_r - \frac{2\pi}{3})v_b + \sin(\theta - \frac{4\pi}{3})v_c\right] - v_q \quad (20)$$

$$0 = \frac{1}{3}(v_a + v_b + v_c) - v_o \quad (21)$$

$$0 = \frac{\sqrt{2}}{3}\left[\cos(\theta_r)i_a + \cos(\theta_r - \frac{2\pi}{3})i_b + \cos(\theta_r - \frac{4\pi}{3})i_c\right] - i_d \quad (22)$$

$$0 = \frac{\sqrt{2}}{3}\left[\sin(\theta_r)i_a + \sin(\theta_r - \frac{2\pi}{3})i_b + \sin(\theta_r - \frac{4\pi}{3})i_c\right] - i_q \quad (23)$$

$$0 = \frac{1}{3}(i_a + i_b + i_c) - i_o \quad (24)$$

$$0 = L_{dd} i_d + L_{df} i_f - \psi_d \quad (25)$$

$$0 = L_{qq} i_q + L_{qq1} i_{q1} - \psi_q \quad (26)$$

$$0 = L_{ff} i_f + L_{df} i_d - \psi_f \quad (27)$$

$$0 = L_{q1q1} i_{q1} + L_{qq1} i_q - \psi_{q1} \quad (28)$$

$$0 = L_{oo} i_o - \psi_o \quad (29)$$

$$\frac{1}{\omega_N} \frac{d\psi_d}{dt} = -R_a i_d - \psi_q - v_d \quad (30)$$

$$\frac{1}{\omega_N} \frac{d\psi_q}{dt} = -R_a i_q + \psi_d - v_q \quad (31)$$

$$\frac{1}{\omega_N} \frac{d\psi_f}{dt} = -R_f i_f + V_f \quad (32)$$

$$\frac{1}{\omega_N} \frac{d\psi_{q1}}{dt} = -R_{q1} i_{q1} \quad (33)$$

$$\frac{1}{\omega_N} \frac{d\psi_o}{dt} = -R_a i_o - v_o \quad (34)$$

Model in compact form

With a proper reordering of equations and states, the model can be rewritten in compact form as:

$$(1/\omega_N)\,\dot{\boldsymbol{x}} = \boldsymbol{A}_{xx}\boldsymbol{x} + \boldsymbol{A}_{xy}\boldsymbol{y} + \boldsymbol{u} \tag{35}$$

$$\boldsymbol{0} = \boldsymbol{A}_{yx}\boldsymbol{x} + \boldsymbol{A}_{yy}\boldsymbol{y} \tag{36}$$

where:

$$\begin{aligned}
\boldsymbol{x} &= [\, i_a \; i_b \; i_c \; \psi_d \; \psi_q \; \psi_f \; \psi_{q1} \; \psi_o \,]^T \\
\boldsymbol{y} &= [\, v_a \; v_b \; v_c \; v_d \; v_q \; v_o \; i_d \; i_q \; i_o \; i_f \; i_{q1} \,]^T \\
\boldsymbol{u} &= [\, -\frac{e_a}{L_e} \;\; -\frac{e_b}{L_e} \;\; -\frac{e_c}{L_e} \;\; 0 \;\; 0 \;\; V_f \;\; 0 \; 0 \,]^T
\end{aligned}$$

Numerical solution of the DAEs

Let k denote the discrete time ($k = 0, 1, 2, \ldots$), and h the time step size.

A popular numerical integration formula is the *Trapezoidal Method* :

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \frac{h}{2}\left(\dot{\boldsymbol{x}}_{k+1} + \dot{\boldsymbol{x}}_k\right)$$

Replacing $\dot{\boldsymbol{x}}_{k+1}$ by its expression (35) :

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \frac{h}{2}\omega_N \boldsymbol{A}_{xx}\boldsymbol{x}_{k+1} + \frac{h}{2}\omega_N \boldsymbol{A}_{xy}\boldsymbol{y}_{k+1} + \frac{h}{2}\omega_N \boldsymbol{u}_{k+1} + \frac{h}{2}\dot{\boldsymbol{x}}_k$$

Dividing by $\frac{h\omega_N}{2}$ and rearranging the various terms :

$$\left[\boldsymbol{A}_{xx} - \frac{2}{h\omega_N}\boldsymbol{I}\right]\boldsymbol{x}_{k+1} + \boldsymbol{A}_{xy}\boldsymbol{y}_{k+1} = -\frac{2}{h\omega_N}\boldsymbol{x}_k - \frac{1}{\omega_N}\dot{\boldsymbol{x}}_k - \boldsymbol{u}_{k+1} \qquad (37)$$

where $\boldsymbol{I}$ is the unit matrix of same dimension as $\boldsymbol{x}$.

On the other hand, from Eq. (36) we have :

$$\boldsymbol{A}_{yx}\boldsymbol{x}_{k+1} + \boldsymbol{A}_{yy}\boldsymbol{y}_{k+1} = \boldsymbol{0} \tag{38}$$

Grouping Eqs. (37) and (38), the linear system to solve at each time step is :

$$\begin{bmatrix} \boldsymbol{A}_{xx} - \frac{2}{h\omega_N}\boldsymbol{I} & \boldsymbol{A}_{xy} \\ \boldsymbol{A}_{yx} & \boldsymbol{A}_{yy} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k+1} \\ \boldsymbol{y}_{k+1} \end{bmatrix} = \begin{bmatrix} -\frac{2}{h\omega_N}\boldsymbol{x}_k - \frac{1}{\omega_N}\dot{\boldsymbol{x}}_k - \boldsymbol{u}_{k+1} \\ \boldsymbol{0} \end{bmatrix} \tag{39}$$

Numerical example and comments on the results

Network and machine data

$f_N = 50$ Hz
$L_e = 0.20$ pu $\quad R_e = 0.01$ pu
$Ra = 0.005$ pu
$L_{dd} = 2.4$ pu $\quad L_{df} = 2.2$ pu $\quad L_{ff} = 2.42$ pu
$L_{qq} = 2.4$ pu $\quad L_{qq1} = 2.2$ pu $\quad L_{q1q1} = 2.2512$ pu
$R_f = 0.0011$ pu $\quad R_{q1} = 0.0239$ pu
$L_{oo} = 0.1$ pu

Initial operating point

$P = 0.5$ pu
$Q = 0.1$ pu
$\bar{V}_a = 1.000$ pu $\angle 0$

A MATLAB script to simulate this system is available in `emt.m` and `linmodel.m`.

Simulation results

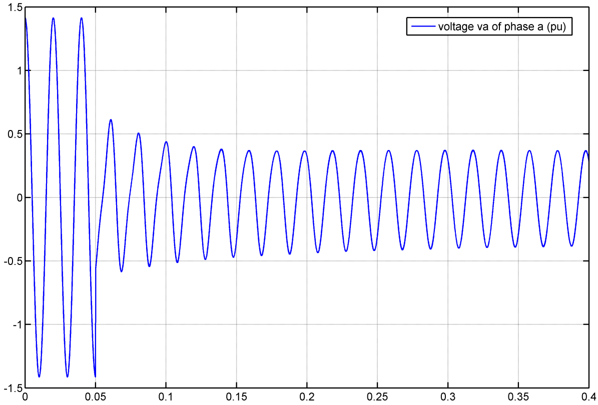
A three-phase short-circuit is simulated by setting E to zero at $t = 0.05$ s.

Important remark

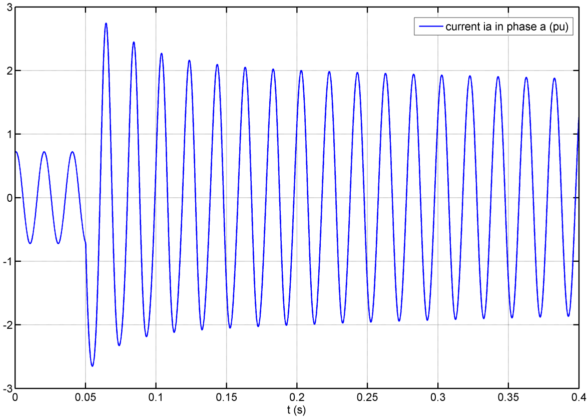
The fault is *not* cleared in order to show the various time constants present in the current evolution.

However, in practice:

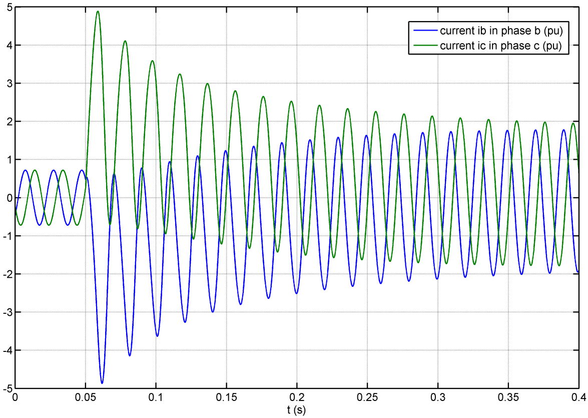
- the fault must be cleared fast enough, e.g. after 5 - 10 cycles (0.1 - 0.2 s)
- beyond that time, the model is no longer valid:
  - rotor speed would not remain constant
  - v_f would be adjusted by the Automatic Voltage Regulator
  - etc.



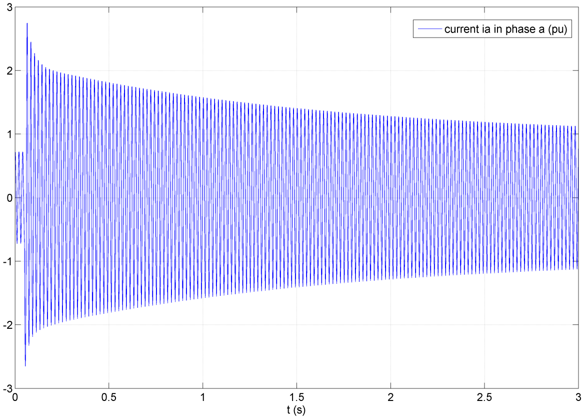
- decrease of AC voltage magnitude under the effect of the fault
- presence of a residual voltage due to some emf inside the generator
- remark:
  - such an emf does not exist in generators connected to the network through power electronic interfaces (dispersed generation in MV distribution grid)
  - the latter do not participate to the short-circuit capacity!



- increase of amplitude of alternating current under the effect of the fault
- the envelop of the current wave varies with time (more details in the sequel)
- presence of a small *aperiodic* or *unidirectional* or *DC* component
  - typical of transients in an RL circuit due to switchings
  - much more visible in the other two phases: see next slide



- the magnitude of the aperiodic components decrease with a time constant $\simeq 0.10$ s (in this example)
- the aperiodic components are not the same in all three phases, because the rotor is not in the same position with respect to each stator winding
- once they have vanished, the three phase currents become again sinusoidal and balanced



- the magnitude of the alternating component of i_a shows two time constants:
  - a short one (a few cycles), resulting in a slightly higher initial amplitude of the current: of subtransient type - caused by damper winding $q1$
  - a much longer one ($\simeq$ 1.5 s in this example): of transient type - caused by field winding f
- the current that the breakers have to interrupt is much higher than the one which would prevail in steady-state !
- the machine behaves initially as if it had a smaller internal reactance

Magnetic fields

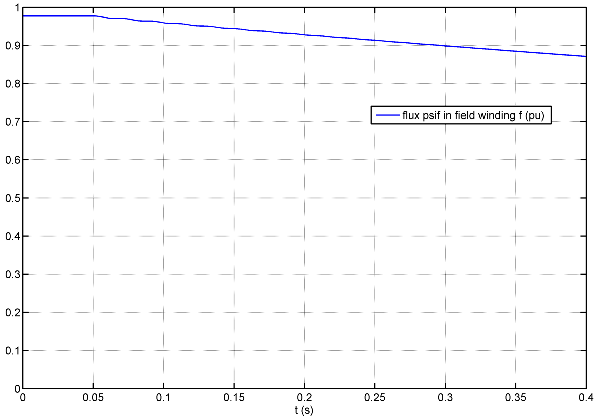
The alternating components of the stator currents i_a, i_b and i_c

- are shifted by $\pm 2\pi/3$ rad. They create a magnetic field H_{AC} which rotates at the angular speed ω_N
- this field is fixed with respect to the rotor windings
- under the effect of the fault, the amplitude if i_a, i_b and i_c increases significantly. So does the magnetic field H_{AC}
- this induces aperiodic current components in the rotor windings.

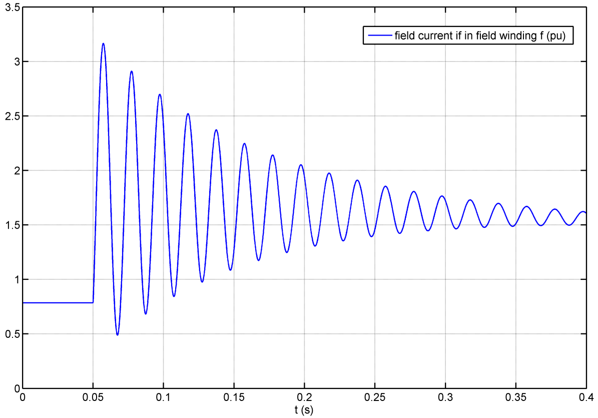
The aperiodic components of the stator currents i_a, i_b and i_c

- create a magnetic field H_{DC} which is fixed with respect to the stator
- hence it rotates at angular speed ω_N with respect to the rotor windings
- this induces alternating components of angular frequency ω_N in the rotor windings.

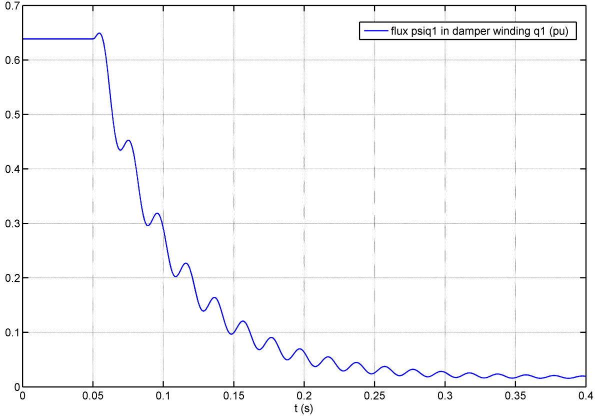
This is confirmed by the plots in the next slides.



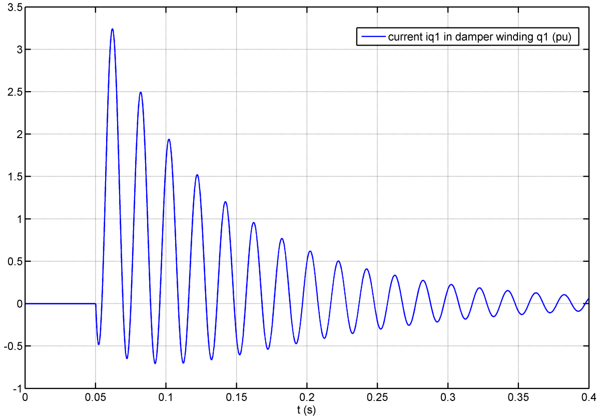
- the flux linkage ψ_f in the field winding changes very little (large "magnetic inertia") in spite of the large increase of stator currents !
- large "magnetic inertia" due to the long time constant $L_{ff}/R_f (= 7$ s in this example)



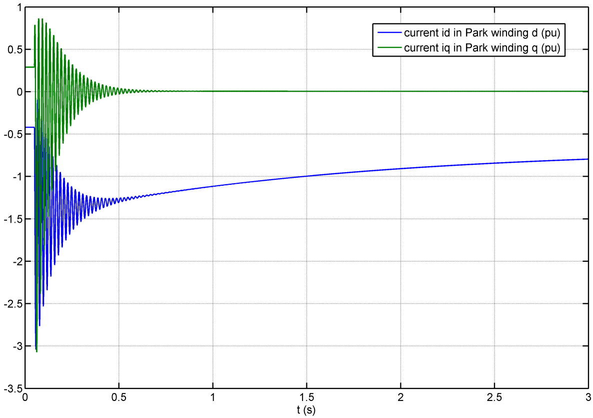
- Lenz law: additional current components appear in the field winding in order to keep ψ_f (almost) constant
- the oscillatory component is due to the magnetic field H_{DC}
  - check: time constant of decay = time constant of aperiodic component of stator currents $\simeq$ 0.10 s
- the aperiodic component is due to the magnetic field H_{AC}
  - time constant $\simeq$ 1.5 s : see slide # 18



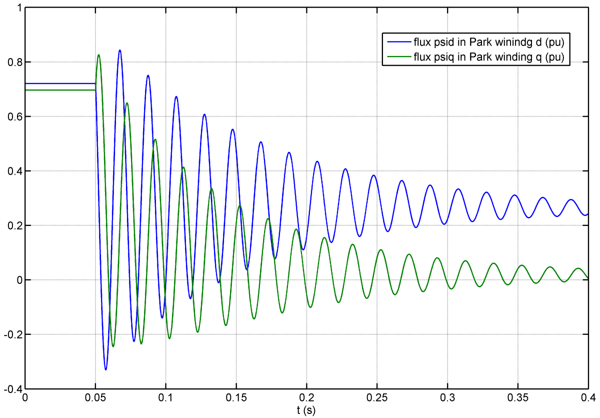
- the flux linkage in the damper winding $q1$ is comparatively more “volatile”
- indeed, the field and the damper windings are constructively very different: field coil vs. damper bars in rotor slots



- the damper current i_{q1} has a zero initial (and final) value
- the oscillatory component is due to the magnetic field H_{DC}
- the aperiodic component decreases much faster than the aperiodic component of i_f
- it corresponds to the initial, fast decaying, increment of the stator current amplitude (see slides # 16 and 18)



- the oscillatory component of i_d (resp. i_q) corresponds to the oscillatory component of i_f (resp. i_{q1}) which lies on the same axis
- it can be shown that i_q goes to almost zero due to the predominantly inductive nature of the short-circuit
- the aperiodic component of i_d evolves with the long time constant observed for the aperiodic component of i_f



- fluxes ψ_d and ψ_q in Park windings vary comparatively much faster
- since i_q and i_{q1} tend to zero, so does ψ_q.