*ELEC0047 - Power system dynamics, control and stability*

# Long-term voltage stability : transmission aspects

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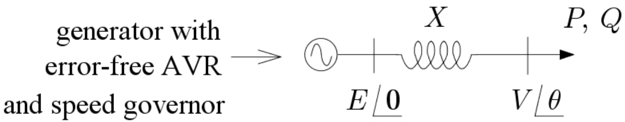
*Voltage instability results from the inability of the combined transmission and generation system to provide the power requested by loads*

# One-generator one-load system in steady-state operation




Complex power absorbed by the load:

$$S = P + jQ = \bar{V}\bar{I}^{\star} = \bar{V}\frac{\bar{E}^{\star} - \bar{V}^{\star}}{-jX} = \frac{j}{X}(EV\cos\theta + jEV\sin\theta - V^2)$$

Power flow equations:

$$P = -\frac{E\,V}{X}\sin\theta \qquad (1)$$

$$Q = -\frac{V^2}{X} + \frac{E\,V}{X}\cos\theta \qquad (2)$$

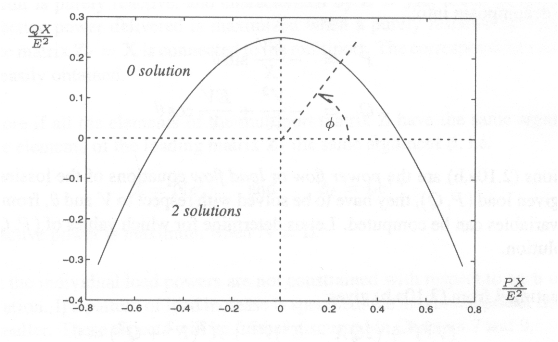
After eliminating $\theta$:

$$\left(V^2\right)^2 + (2QX - E^2)V^2 + X^2(P^2 + Q^2) = 0 \qquad (3)$$

# Feasible region in load power space

To have (at least) one solution:

$$-\left(\frac{P\,X}{E^2}\right)^2 - \frac{Q\,X}{E^2} + 0.25 \geq 0 \tag{4}$$




- any $P$ can be reached provided $Q$ is adjusted (but $V$ may be unacceptable !)
- dissymmetry between $P$ and $Q$ due to reactive transmission impedance
- locus symmetric w.r.t. $Q$ axis; not true any longer when some transmission resistance is included

# Maximum load power under constant power factor

Under the given load power factor $\cos\phi$ : $\qquad Q = P \tan\phi$

Substituting in (4) gives:

$$P^2 + \frac{E^2}{X}\tan\phi\, P - \frac{E^4}{4X^2} = 0$$

from which one obtains:

$$P_{max} = \frac{\cos\phi}{1+\sin\phi}\frac{E^2}{2X} \qquad Q_{max} = \frac{\sin\phi}{1+\sin\phi}\frac{E^2}{2X} \qquad V_{maxP} = \frac{E}{\sqrt{2}\sqrt{1+\sin\phi}}$$

Particular cases:

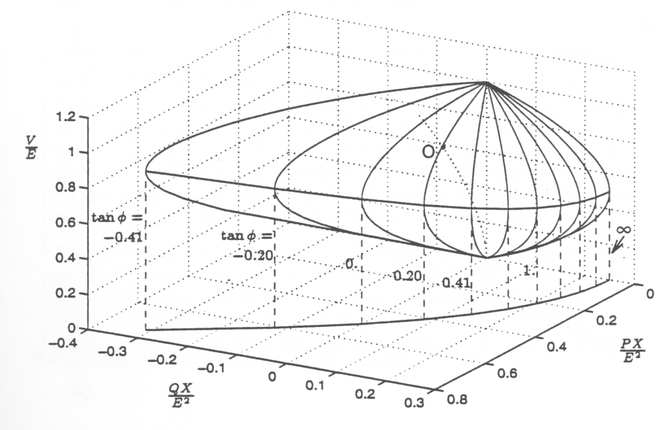
$$\cos\phi = 1: \qquad P_{max} = \frac{E^2}{2X} \qquad Q_{max} = 0 \qquad V_{maxP} = \frac{E}{\sqrt{2}}$$

$$\cos\phi = 0: \qquad P_{max} = 0 \qquad Q_{max} = \frac{E^2}{4X} \qquad V_{maxP} = \frac{E}{2}$$

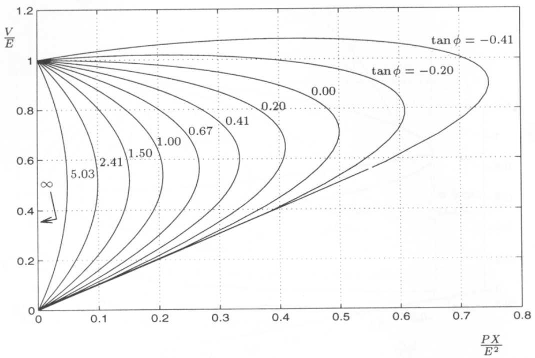
# Load voltage as a function of load power

Solving (3) for $V^2$ and taking the square root:

$$V = \sqrt{\frac{E^2}{2} - QX \pm \sqrt{\frac{E^4}{4} - X^2P^2 - XQE^2}}$$

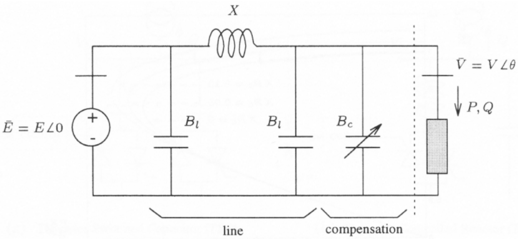


# PV curves




- for a given power :
  
  1 solution with "high" voltage and "low" current (normal operating point)
  
  1 solution with "low" voltage and "high" current
- compensating the load increases the maximum power but the "critical" voltage approaches normal values !
- similar curves: QV or SV under constant $\tan\phi$, QV under constant $P$, etc.

# Effect of line capacitance and/or shunt compensation




Thevenin equivalent seen by the load:

$$E_{th} = \frac{E}{1-(B_c+B_l)X} \qquad X_{th} = \frac{X}{1-(B_c+B_l)X}$$

Maximum deliverable power under power factor $\cos\phi$:

$$P_{max} = \frac{\cos\phi}{1+\sin\phi}\,\frac{E_{th}^2}{2X_{th}} = \frac{1}{1-(B_c+B_l)X}\,\frac{\cos\phi}{1+\sin\phi}\,\frac{E^2}{2X}$$

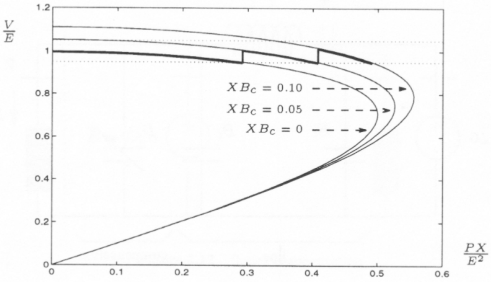
corresponding load voltage:

$$V_{maxP} = \frac{E_{th}}{\sqrt{2}\sqrt{1+\sin\phi}} = \frac{1}{1-(B_c+B_l)X}\,\frac{E}{\sqrt{2}\sqrt{1+\sin\phi}}$$

# Effect of variable shunt compensation

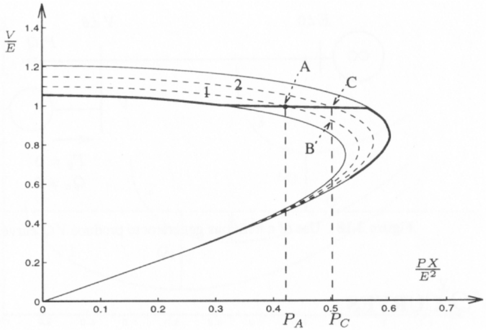
$B_l = 0, \; B_c$ varied

shunt compensation adjusted to keep voltage in an interval




smooth change in compensation by SVC at load bus

$V$ almost constant as long as SVC is within limits



# Power flow Jacobian and sensitivities

## Jacobian of power flow equations

The power flow Eqs. (1, 2) are rewritten as :

$$\frac{E\,V}{X}\sin\theta + P = 0 \tag{5}$$

$$\frac{V^2}{X} - \frac{E\,V}{X}\cos\theta + Q = 0 \tag{6}$$

A linearization of these equations for small variations $\Delta P$ and $\Delta Q$ gives :

$$\underbrace{\begin{bmatrix} \frac{EV}{X}\cos\theta & \frac{E}{X}\sin\theta \\ \frac{EV}{X}\sin\theta & \frac{2V}{X} - \frac{E}{X}\cos\theta \end{bmatrix}}_{\boldsymbol{J}} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = -\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{7}$$

where $\boldsymbol{J}$ is the Jacobian matrix of the power flow equations with respect to the state variables $V$ and $\theta$.

## Singularity of the Jacobian at the maximum load point

The maximum load point (under constant power factor) is the solution of :

$$
\begin{aligned}
& max_{\rho,V,\theta} \quad \rho \\
\text{subject to :} \qquad & \frac{E\,V}{X}\sin\theta + \rho P^o = 0 \\
& \frac{V^2}{X} - \frac{E\,V}{X}\cos\theta + \rho Q^o = 0
\end{aligned}
$$

Consider the Lagrangian :

$$
\mathcal{L} = \rho + \mu_p\left(\frac{E\,V}{X}\sin\theta + \rho P^o\right) + \mu_q\left(\frac{V^2}{X} - \frac{E\,V}{X}\cos\theta + \rho Q^o\right)
$$

The ((Karush-)Kuhn-Tucker) necessary conditions of optimality give :

$$
\frac{\partial \mathcal{L}}{\partial \rho} = 0 \quad \Leftrightarrow \quad 1 + \mu_p P^o + \mu_q Q^o = 0 \tag{8}
$$

$$
\frac{\partial \mathcal{L}}{\partial \theta} = 0 \quad \Leftrightarrow \quad \mu_p \frac{EV}{X}\cos\theta + \mu_q \frac{EV}{X}\sin\theta = 0 \tag{9}
$$

$$
\frac{\partial \mathcal{L}}{\partial V} = 0 \quad \Leftrightarrow \quad \mu_p \frac{E}{X}\sin\theta + \mu_q\left(\frac{2V}{X} - \frac{E}{X}\cos\theta\right) = 0 \tag{10}
$$

$$\frac{\partial \mathcal{L}}{\partial \mu_p} = 0 \quad \Leftrightarrow \quad \frac{E\,V}{X}\sin\theta + \rho P^o = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_q} = 0 \quad \Leftrightarrow \quad \frac{V^2}{X} - \frac{E\,V}{X}\cos\theta + \rho Q^o = 0 \tag{12}$$

Eqs. (9, 10) can be rewritten as :

$$\left[\begin{array}{cc} \mu_p & \mu_q \end{array}\right] \underbrace{\left[\begin{array}{cc} \frac{EV}{X}\cos\theta & \frac{E}{X}\sin\theta \\ \frac{EV}{X}\sin\theta & \frac{2V}{X} - \frac{E}{X}\cos\theta \end{array}\right]}_{\boldsymbol{J}} = \left[\begin{array}{cc} 0 & 0 \end{array}\right]$$

while Eq. (8) shows that :

$$\left[\begin{array}{c} \mu_p \\ \mu_q \end{array}\right] \neq \left[\begin{array}{c} 0 \\ 0 \end{array}\right]$$

Hence, it can be concluded that *the Jacobian $\boldsymbol{J}$ of the power flow equations is singular at the maximum load power point.*

## Sensitivity of the generator reactive power to the load reactive power

*How many additional Mvar must be produced if the load consumes 1 Mvar more ?*

The reactive power generation is given by :

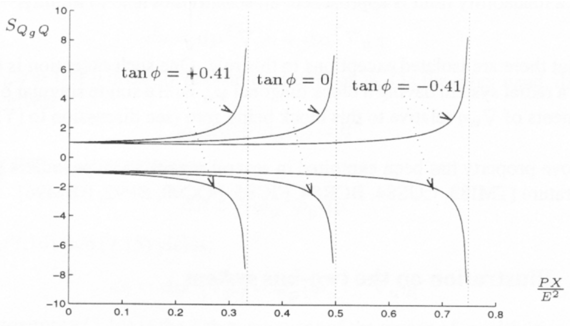
$$Q_g = \frac{E^2}{X} - \frac{EV}{X}\cos\theta$$

A linearization of this equation for small variations $\Delta P$ and $\Delta Q$ gives :

$$\begin{aligned}
\Delta Q_g &= \frac{EV}{X}\sin\theta\;\Delta\theta - \frac{E}{X}\cos\theta\;\Delta V \\
&= \left[\begin{array}{cc} \frac{EV}{X}\sin\theta & -\frac{E}{X}\cos\theta \end{array}\right] \left[\begin{array}{c} \Delta\theta \\ \Delta V \end{array}\right] \\
&= -\left[\begin{array}{cc} \frac{EV}{X}\sin\theta & -\frac{E}{X}\cos\theta \end{array}\right] \boldsymbol{J}^{-1} \left[\begin{array}{c} \Delta P \\ \Delta Q \end{array}\right]
\end{aligned} \tag{13}$$

The sought sensitivity is $S_{Q_g Q} = \dfrac{\Delta Q_g}{\Delta Q}$.

Eq. (13) shows that, as the maximum load power point is approached, the sensitivity tends to infinity.



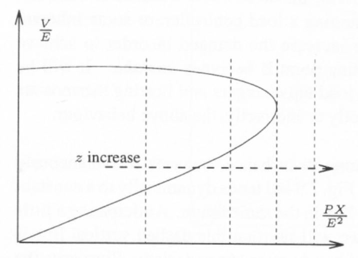

It can also be shown that the sensitivity changes from positive to negative as the operating point moves from the upper to the lower part of a PV curve.

This allows identifying on which part of the PV curve the oper. point is located.

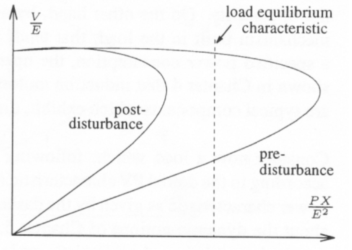
# First glance at instability mechanisms

Assume that the load behaves as constant power in steady-state

$$P = zP_o \qquad Q = zQ_o$$




When $z$ increases, the load characteristic changes until it eventually does not intersect the network characteristic. Equilibrium is lost at the *loadability limit*.




The large disturbance causes the network characteristic to shrink so drastically that the post-disturbance network characteristic does no longer intersect the (unchanged) load characteristic