ELEC0029 - Electric power systems analysis and operation

Analysis of unbalanced systems : the symmetrical components

Thierry Van Cutsem
t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

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Recall: the operator a

$$a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

with obviously:

$$a^2 = e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}} = a^{\star} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$1 + a + a^2 = 0$$

Symmetrical components: definition

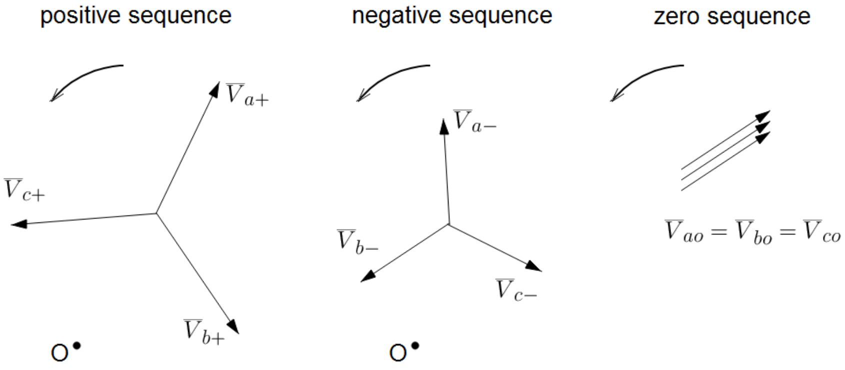
Consider an electrical circuit:

- made up of three phases
- operating in sinusoidal steady-state
- operating in unbalanced conditions.

A set of three unbalanced phasors can be decomposed into the sum of:

- three phasors making up a *positive* (or *direct*) *sequence*
- three phasors making up an *negative sequence*
- three phasors making up a *zero sequence*

In what follows, we consider voltages but all derivations equally apply to currents.



- positive sequence: three rotating vectors, of same magnitude, shifted by 120^o which an observer sees passing in the order $a, b, c, a, b, c \ldots$
  $\rightarrow$ denoted $+$
- negative sequence: three rotating vectors, of same magnitude, shifted by 120^o which an observer sees passing in the order $a, c, b, a, c, b \ldots$
  $\rightarrow$ denoted $-$
- zero sequence: three rotating vectors, of same magnitude and in phase
  $\rightarrow$ denoted o.

How obtain the above mentioned decomposition:

$$\begin{aligned}
\overline{V}_a &= \overline{V}_{a+} + \overline{V}_{a-} + \overline{V}_{ao} = \overline{V}_+ + \overline{V}_- + \overline{V}_o \\
\overline{V}_b &= \overline{V}_{b+} + \overline{V}_{b-} + \overline{V}_{bo} = a^2\,\overline{V}_+ + a\,\overline{V}_- + \overline{V}_o \\
\overline{V}_c &= \overline{V}_{c+} + \overline{V}_{c-} + \overline{V}_{co} = a\,\overline{V}_+ + a^2\,\overline{V}_- + \overline{V}_o
\end{aligned}$$

In matrix form:

$$\underbrace{\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix}}_{\overline{\boldsymbol{V}}_T} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}}_{\boldsymbol{T}} \underbrace{\begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix}}_{\overline{\boldsymbol{V}}_F}$$

$\overline{V}_+$: *positive-sequence component* $\qquad$ $\overline{V}_-$: *negative-sequence component*

$\overline{V}_o$: *zero-sequence component*

("of phase a" implied)

$=$ *symmetrical components* or *Fortescue components* of $\overline{V}_a, \overline{V}_b, \overline{V}_c$

The same transformation by matrix $\boldsymbol{T}$ applies to currents.

Inverse transformation

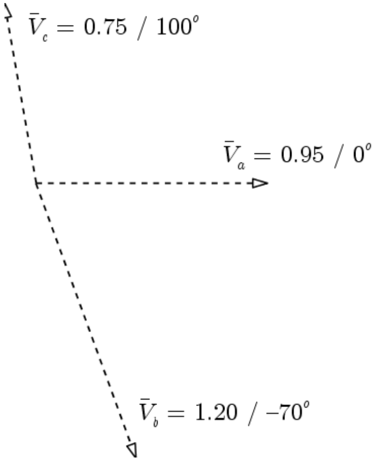
$$\begin{aligned} \overline{\boldsymbol{V}}_F &= \boldsymbol{T}^{-1}\overline{\boldsymbol{V}}_T \\ \begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix} &= \boldsymbol{T}^{-1} \begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \overline{\boldsymbol{I}}_F &= \boldsymbol{T}^{-1}\overline{\boldsymbol{I}}_T \\ \begin{bmatrix} \overline{I}_+ \\ \overline{I}_- \\ \overline{I}_o \end{bmatrix} &= \boldsymbol{T}^{-1} \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix} \end{aligned}$$

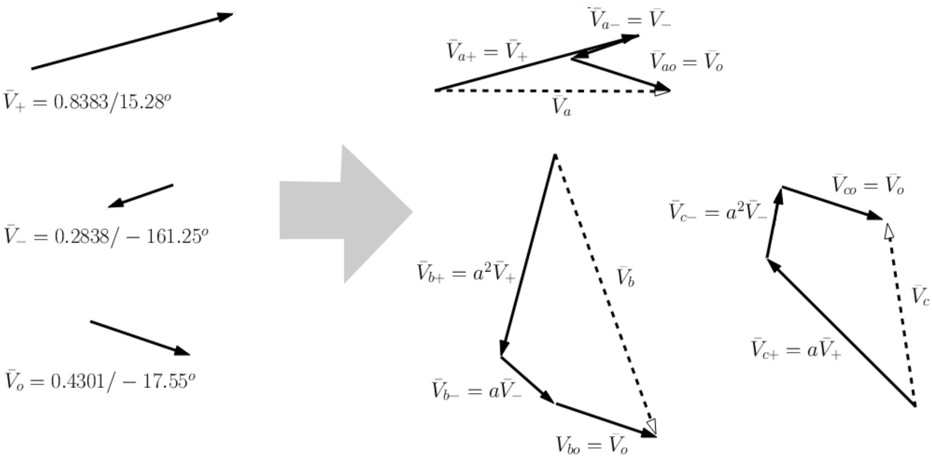
with:

$$\boldsymbol{T}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3} \left[\boldsymbol{T}^{\star}\right]^T$$

Numerical example



$$\begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.95\angle 0^o \\ 1.20\angle -70^o \\ 0.75\angle 100^o \end{bmatrix} = \begin{bmatrix} 0.8383\angle 15.28^o \\ 0.2838\angle -161.25^o \\ 0.4301\angle -17.55^o \end{bmatrix}$$



$$
\begin{aligned}
\overline{V}_a &= \overline{V}_+ + \overline{V}_- + \overline{V}_o \\
\overline{V}_b &= a^2\,\overline{V}_+ + a\,\overline{V}_- + \overline{V}_o \\
\overline{V}_c &= a\,\overline{V}_+ + a^2\,\overline{V}_- + \overline{V}_o
\end{aligned}
$$

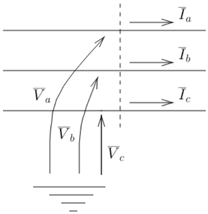
Remarks

1. Obvious feature: in balanced three-phase operation, voltages and currents only have positive-sequence components; the negative-sequence and the zero-sequence components are zero.

2. Anglo-Saxon vs. French terminology :

symmetrical components		composantes symétriques	
positive-sequence	denoted $+$ or 1	directe	notée d
negative-sequence	denoted $-$ or 2	inverse	notée i
zero-sequence	denoted o	homopolaire	notée o

Powers and symmetrical components



Complex power flowing through the three phases:

$$\begin{aligned} S &= \overline{V}_a\overline{I}_a^\star + \overline{V}_b\overline{I}_b^\star + \overline{V}_c\overline{I}_c^\star = \overline{\boldsymbol{V}}_T^T\overline{\boldsymbol{I}}_T^\star \\ &= \left[\boldsymbol{T}\overline{\boldsymbol{V}}_F\right]^T\left[\boldsymbol{T}\overline{\boldsymbol{I}}_F\right]^\star = \overline{\boldsymbol{V}}_F^T\boldsymbol{T}^T\boldsymbol{T}^\star\overline{\boldsymbol{I}}_F^\star = 3\,\overline{\boldsymbol{V}}_F^T\boldsymbol{T}^T\left[\boldsymbol{T}^{-1}\right]^T\overline{\boldsymbol{I}}_F^\star = 3\,\overline{\boldsymbol{V}}_F^T\overline{\boldsymbol{I}}_F^\star \\ &= 3\left(\overline{V}_+\overline{I}_+^\star + \overline{V}_-\overline{I}_-^\star + \overline{V}_o\overline{I}_o^\star\right) \end{aligned}$$

The coefficient 3 comes from the fact that $\overline{V}_+\overline{I}_+^\star$ is only one third of the power in the positive sequence, $\overline{V}_-\overline{I}_-^\star$ only one third of the power in negative sequence, and $\overline{V}_o\overline{I}_o^\star$ only one third of the power in the zero sequence

Positive, negative and zero-sequence impedances of a load

$$\begin{aligned} \overline{\boldsymbol{I}}_T &= \boldsymbol{Y}\,\overline{\boldsymbol{V}}_T \\ \boldsymbol{T}\,\overline{\boldsymbol{I}}_F &= \boldsymbol{Y}\,\boldsymbol{T}\,\overline{\boldsymbol{V}}_F \\ \overline{\boldsymbol{I}}_F &= \underbrace{\boldsymbol{T}^{-1}\,\boldsymbol{Y}\,\boldsymbol{T}}_{\boldsymbol{Y}_F}\overline{\boldsymbol{V}}_F \end{aligned}$$

$$\begin{aligned} \overline{\boldsymbol{V}}_T &= \boldsymbol{Z}\,\overline{\boldsymbol{I}}_T \\ \boldsymbol{T}\,\overline{\boldsymbol{V}}_F &= \boldsymbol{Z}\,\boldsymbol{T}\,\overline{\boldsymbol{I}}_F \\ \overline{\boldsymbol{V}}_F &= \underbrace{\boldsymbol{T}^{-1}\,\boldsymbol{Z}\,\boldsymbol{T}}_{\boldsymbol{Z}_F}\overline{\boldsymbol{I}}_F \end{aligned}$$

If $\boldsymbol{Y}$ has full three-phase symmetry:

$$\boldsymbol{Y} = \begin{bmatrix} y_s & y_m & y_m \\ y_m & y_s & y_m \\ y_m & y_m & y_s \end{bmatrix}$$

then $\boldsymbol{Y}_F$ is the diagonal matrix:

$$\begin{bmatrix} y_s - y_m & 0 & 0 \\ 0 & y_s - y_m & 0 \\ 0 & 0 & y_s + 2y_m \end{bmatrix}$$

If $\boldsymbol{Z}$ has full three-phase symmetry:

$$\boldsymbol{Z} = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}$$

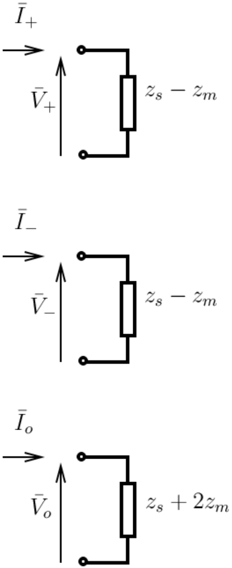
then $\boldsymbol{Z}_F$ is the diagonal matrix:

$$\begin{bmatrix} z_s - z_m & 0 & 0 \\ 0 & z_s - z_m & 0 \\ 0 & 0 & z_s + 2z_m \end{bmatrix}$$

positive-sequence equivalent circuit

negative-sequence equivalent circuit

zero-sequence equivalent circuit



Under the above-mentioned assumption that the load has full three-phase symmetry, the positive, negative and zero-sequence circuits are fully decoupled.

Interpretation of positive-, negative- and zero-sequence impedances

Note. The following interpretations are general and can be extended to the other network components.

The positive-sequence impedance $z_s - z_m$ is the impedance seen in one phase when the load is supplied by positive sequence currents.

$$\text{Proof:} \qquad \bar{\boldsymbol{I}}_T = \begin{bmatrix} \bar{I}_a \\ a^2\bar{I}_a \\ a\bar{I}_a \end{bmatrix} \quad \Rightarrow \quad \bar{\boldsymbol{I}}_F = \boldsymbol{T}^{-1}\bar{\boldsymbol{I}}_T = \begin{bmatrix} \bar{I}_a \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{V}_+ = (z_s - z_m)\bar{I}_+ = (z_s - z_m)\bar{I}_a \qquad \overline{V}_- = (z_s - z_m)\bar{I}_- = 0 \qquad \overline{V}_o = (z_s + 2z_m)\bar{I}_o = 0$$

$$\Rightarrow \quad \overline{V}_a = \overline{V}_+ + \overline{V}_- + \overline{V}_o = \overline{V}_+ = (z_s - z_m)\bar{I}_a \quad \Diamond$$

The positive-sequence impedance is nothing but the *cyclic impedance* defined in the per phase analysis (see course ELEC0014).

The negative-sequence impedance $z_s - z_m$ is the impedance seen in one phase when the load is supplied by negative sequence currents.

$$\text{Proof:} \qquad \bar{\boldsymbol{I}}_T = \begin{bmatrix} \bar{I}_a \\ a\bar{I}_a \\ a^2\bar{I}_a \end{bmatrix} \quad \Rightarrow \quad \bar{\boldsymbol{I}}_F = \boldsymbol{T}^{-1}\bar{\boldsymbol{I}}_T = \begin{bmatrix} 0 \\ \bar{I}_a \\ 0 \end{bmatrix}$$

$$\overline{V}_+ = 0 \qquad \overline{V}_- = (z_s - z_m)\bar{I}_- = (z_s - z_m)\bar{I}_a \qquad \overline{V}_o = 0$$

$$\Rightarrow \quad \overline{V}_a = \overline{V}_+ + \overline{V}_- + \overline{V}_o = \overline{V}_- = (z_s - z_m)\bar{I}_a \quad \Diamond$$

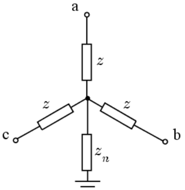
The zero-sequence impedance $z_s + 2z_m$ is the impedance seen in one phase when the load is supplied by zero sequence currents.

$$\text{Proof:} \qquad \bar{\boldsymbol{I}}_T = \begin{bmatrix} \bar{I}_a \\ \bar{I}_a \\ \bar{I}_a \end{bmatrix} \quad \Rightarrow \quad \bar{\boldsymbol{I}}_F = \boldsymbol{T}^{-1}\bar{\boldsymbol{I}}_T = \begin{bmatrix} 0 \\ 0 \\ \bar{I}_a \end{bmatrix}$$

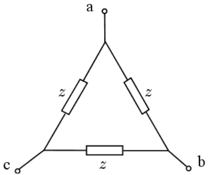
$$\overline{V}_+ = 0 \qquad \overline{V}_- = 0 \qquad \overline{V}_o = (z_s + 2z_m)\bar{I}_o = (z_s + 2z_m)\bar{I}_a$$

$$\Rightarrow \quad \overline{V}_a = \overline{V}_+ + \overline{V}_- + \overline{V}_o = \overline{V}_o = (z_s + 2z_m)\bar{I}_a \quad \Diamond$$

Exercise 1. Consider the three-phase load obtained by assembling three impedances z in a star and connecting the neutral to the ground through an impedance z_n. Show that the positive and negative-sequence impedances are z and the zero-sequence impedance $z + 3z_n$.

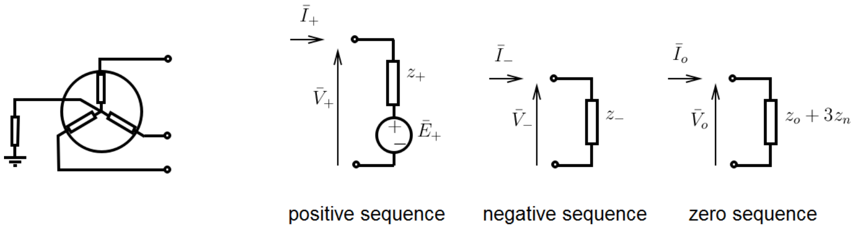


Exercise 2. Consider the three-phase load obtained by assembling three impedances z in a triangle. Show that the positive and negative-sequence impedances are $z/3$ and the zero-sequence impedance is infinite.



Positive, negative and zero-sequence equivalent circuits of a synchronous generator

We assume a star configuration, with an impedance z_n in the neutral.



- By construction, generators present a three-phase symmetry $\Rightarrow$ the positive, negative and zero-sequence equivalent circuits are decoupled (see case of load in slide # 12)
- the neutral impedance appears in the zero-sequence circuit only, and multiplied by 3 (see Exercise 1)
- if the stator windings are connected in triangle, z_n is infinite (see Exercise 2).

Positive-sequence equivalent circuit

- A synchronous generator aims at producing positive-sequence voltages; hence, an e.m.f. ($\bar{E}_+$) appears in the positive-sequence equivalent circuit *only*
- the positive-sequence equivalent circuit is the equivalent circuit already known from the analysis of the generator in balanced three-phase operation
- the impedance z_+ is the stator resistance in series with a reactance that depends on the time interval considered:
  - unbalanced permanent operation: consider the synchronous reactance X
  - immediately after a short-circuit: consider the subtransient reactance X''
- in case of short-circuit analysis, the e.m.f. $\bar{E}_+$ behind the reactance X'' is assumed constant and is determined from the pre-fault operating conditions (see lecture "Behaviour of synchronous machine during a short-circuit")

Negative-sequence impedance

Recall from course ELEC0014: in the air gap, at an angle φ of the axis of phase a, the magnitude of the magnetic field produced by the three phases is:

$$H_{3\phi}(\varphi) = k i_a \cos\varphi + k i_b \cos(\varphi - \frac{2\pi}{3}) + k i_c \cos(\varphi - \frac{4\pi}{3})$$

If the stator windings are supplied with positive-sequence three-phase currents:

$$\begin{aligned} H_{3\phi}(\varphi) &= \sqrt{2}kI \Big[\cos(\omega t + \psi)\cos\varphi + \cos(\omega t + \psi - \frac{2\pi}{3})\cos(\varphi - \frac{2\pi}{3}) + \\ & \quad + \cos(\omega t + \psi - \frac{4\pi}{3})\cos(\varphi - \frac{4\pi}{3})\Big] = \frac{3\sqrt{2}kI}{2}\cos(\omega t + \psi - \varphi) \end{aligned}$$

$\equiv$ equation of a magnetic field rotating in the air gap with angular speed ω.

If the stator windings are supplied with negative-sequence three-phase currents:

$$\begin{aligned} H_{3\phi}(\varphi) &= \sqrt{2}kI \Big[\cos(\omega t + \psi)\cos\varphi + \cos(\omega t + \psi + \frac{2\pi}{3})\cos(\varphi - \frac{2\pi}{3}) + \\ & \quad + \cos(\omega t + \psi + \frac{4\pi}{3})\cos(\varphi - \frac{4\pi}{3})\Big] = \frac{3\sqrt{2}kI}{2}\cos(\omega t + \psi + \varphi) \end{aligned}$$

$\equiv$ equation of a magnetic field rotating in the air gap with angular speed $-\omega$.

- z_- is the impedance seen in one phase, when a negative sequence of currents is injected in the machine ...
- ... after having set v_f to zero (field winding short-circuited)
    - since this voltage creates positive-sequence stator voltages already taken into account in the positive-sequence equivalent circuit
- the negative sequence of currents produces a magnetic field rotating at speed $-\omega$ opposed to that of the rotor
- thus, the generator operates like an induction machine with a slip:

$$s = \frac{-\omega - \omega}{-\omega} = 2$$

- z_- is the impedance seen in one phase of that induction machine with $s = 2$
- $z_- = r_- + jx_-$
  x_- is close to the subtransient reactance X''. Indeed, the stator magnetic field induces currents of angular frequency 2ω in the rotor circuits. These currents tend to maintain a constant flux in the rotor circuits. The lines of the magnetic field are close to those that prevail just after a short-circuit (hence $x_- \simeq X''$).

Zero-sequence impedance

In the air gap, at an angle φ of the axis of phase a, the magnitude of the magnetic field produced by the three phases is:

$$H_{3\phi}(\varphi) = ki_a \cos\varphi + ki_b \cos(\varphi - \frac{2\pi}{3}) + ki_c \cos(\varphi - \frac{4\pi}{3})$$

If the stator windings are supplied with zero-sequence three-phase currents:

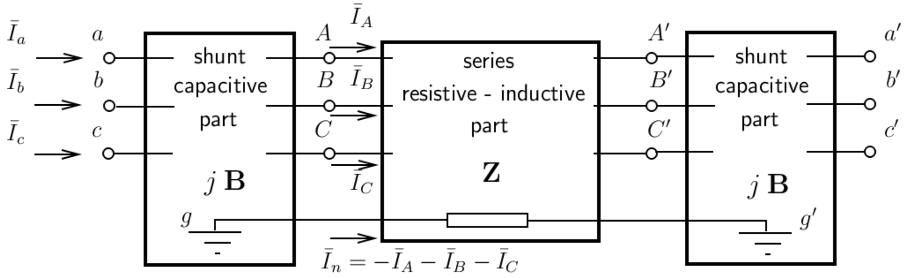
$$\begin{aligned} H_{3\phi}(\varphi) &= \sqrt{2}kI \Big[\cos(\omega t + \psi)\cos\varphi + \cos(\omega t + \psi)\cos(\varphi - \frac{2\pi}{3}) + \\ &\quad + \cos(\omega t + \psi)\cos(\varphi - \frac{4\pi}{3})\Big] \\ &= \sqrt{2}kI\cos(\omega t + \psi)\Big[\cos\varphi + \cos(\varphi - \frac{2\pi}{3}) + \cos(\varphi - \frac{4\pi}{3})\Big] = 0 \end{aligned}$$

i.e. there is no magnetic field in the air gap.

- z_o is the impedance seen in one phase, when a zero sequence of currents is injected in the machine ...
- ... after having set v_f to zero (field winding short-circuited)
  - since this voltage creates positive-sequence stator voltages already taken into account in the positive-sequence equivalent circuit
- zero-sequence stator currents produce no magnetic field in the air gap
- the magnetic flux in one stator winding is produced by:
  - the current flowing in that winding
  - taking into account only the lines of magnetic field which cross that winding but do not cross the air gap (in which they are canceled by the fields of the other stator windings)
  - this is the leakage flux of the stator winding
- $z_o = R_a + jX_\ell$
  R_a is the stator resistance
  X_ℓ is the stator leakage reactance; it is much smaller than the synchronous reactance: in the range $0.1 - 0.2$ pu.

Positive, negative and zero-sequence equiv circuits of a line

Also applies to cables



- the return current $-\bar{I}_n = \bar{I}_A + \bar{I}_B + \bar{I}_C$ may be nonzero [1]
- this current flows partly in the ground and mainly in the shield wires
- the latter are magnetically coupled with the (a, b, c) phase wires
- the various points on the ground are no longer at the same voltage
- in what follows the voltage of each phase is referred to the "local" ground (i.e. the ground of the same switching station).

[1] this depends on the grounding of the other elements: see further in this chapter

Treatment of the series part of the line model

We have:

$$\overline{V}_A - \overline{V}_g = (\overline{V}_{A'} - \overline{V}_{g'}) + (\overline{V}_A - \overline{V}_{A'}) + (\overline{V}_{g'} - \overline{V}_g)$$

with:

$$\begin{aligned}
\overline{V}_A - \overline{V}_{A'} &= Z_{aa}\overline{I}_A + Z_{ab}\overline{I}_B + Z_{ac}\overline{I}_C + Z_{an}\overline{I}_n \\
&= (Z_{aa} - Z_{an})\overline{I}_A + (Z_{ab} - Z_{an})\overline{I}_B + (Z_{ac} - Z_{an})\overline{I}_C
\end{aligned}$$

$$\begin{aligned}
\overline{V}_{g'} - \overline{V}_g &= -\left(Z_{an}\overline{I}_A + Z_{bn}\overline{I}_B + Z_{cn}\overline{I}_C + Z_{nn}\overline{I}_n\right) \\
&= (Z_{nn} - Z_{an})\overline{I}_A + (Z_{nn} - Z_{bn})\overline{I}_B + (Z_{nn} - Z_{cn})\overline{I}_C
\end{aligned}$$

and, hence:

$$\begin{aligned}
\overline{V}_A - \overline{V}_g &= (\overline{V}_{A'} - \overline{V}_{g'}) \\
&\quad + (Z_{aa} + Z_{nn} - 2Z_{an})\overline{I}_A + (Z_{ab} + Z_{nn} - Z_{an} - Z_{bn})\overline{I}_B \\
&\quad + (Z_{ac} + Z_{nn} - Z_{an} - Z_{cn})\overline{I}_C
\end{aligned}$$

with similar expressions for phases b and c.

In matrix form:

$$\begin{bmatrix} \overline{V}_A - \overline{V}_g \\ \overline{V}_B - \overline{V}_g \\ \overline{V}_C - \overline{V}_g \end{bmatrix} = \boldsymbol{Z} \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix} + \begin{bmatrix} \overline{V}_{A'} - \overline{V}_{g'} \\ \overline{V}_{B'} - \overline{V}_{g'} \\ \overline{V}_{C'} - \overline{V}_{g'} \end{bmatrix}$$

with:

$$\boldsymbol{Z} = \begin{bmatrix} Z_{aa} + Z_{nn} - 2Z_{an} & Z_{ab} + Z_{nn} - Z_{an} - Z_{bn} & Z_{ac} + Z_{nn} - Z_{an} - Z_{cn} \\ Z_{ab} + Z_{nn} - Z_{an} - Z_{bn} & Z_{bb} + Z_{nn} - 2Z_{bn} & Z_{bc} + Z_{nn} - Z_{bn} - Z_{cn} \\ Z_{ac} + Z_{nn} - Z_{an} - Z_{cn} & Z_{bc} + Z_{nn} - Z_{bn} - Z_{cn} & Z_{cc} + Z_{nn} - 2Z_{cn} \end{bmatrix}$$

Passing to the symmetric components:

$$\boldsymbol{T} \begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix}_{ABC} = \boldsymbol{Z}\,\boldsymbol{T} \begin{bmatrix} \overline{I}_+ \\ \overline{I}_- \\ \overline{I}_o \end{bmatrix}_{ABC} + \boldsymbol{T} \begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix}_{A'B'C'}$$

$$\begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix}_{ABC} = \boldsymbol{T}^{-1}\boldsymbol{Z}\boldsymbol{T} \begin{bmatrix} \overline{I}_+ \\ \overline{I}_- \\ \overline{I}_o \end{bmatrix}_{ABC} + \begin{bmatrix} \overline{V}_+ \\ \overline{V}_- \\ \overline{V}_o \end{bmatrix}_{A'B'C'}$$

If the line presents the following symmetries[2]:

$$Z_{ab} = Z_{ac} = Z_{bc}$$
$$Z_{an} = Z_{bn} = Z_{cn}$$
$$Z_{aa} = Z_{bb} = Z_{cc}$$

then $\boldsymbol{Z}$ has full three-phase symmetry:

$$\boldsymbol{Z} = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}$$

and we have:

$$\boldsymbol{T}^{-1}\boldsymbol{Z}\boldsymbol{T} = \begin{bmatrix} z_s - z_m & 0 & 0 \\ 0 & z_s - z_m & 0 \\ 0 & 0 & z_s + 2z_m \end{bmatrix}$$

with

$$\begin{aligned} z_s - z_m &= Z_{aa} - Z_{ab} \\ z_s + 2z_m &= Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} \end{aligned}$$

[2] this is an approximation: see course ELEC0014

Treatment of the shunt part of the line model

Consider the one located to the left of the series part in slide # 22

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} \bar{I}_A \\ \bar{I}_B \\ \bar{I}_C \end{bmatrix} + j\boldsymbol{B} \begin{bmatrix} \bar{V}_a - \bar{V}_g \\ \bar{V}_b - \bar{V}_g \\ \bar{V}_c - \bar{V}_g \end{bmatrix}$$

$$\bar{V}_a = \bar{V}_A \qquad \bar{V}_b = \bar{V}_B \qquad \bar{V}_c = \bar{V}_C$$

Passing to the symmetric components:

$$\boldsymbol{T} \begin{bmatrix} \bar{I}_+ \\ \bar{I}_- \\ \bar{I}_o \end{bmatrix}_{abc} = \boldsymbol{T} \begin{bmatrix} \bar{I}_+ \\ \bar{I}_- \\ \bar{I}_o \end{bmatrix}_{ABC} + j\boldsymbol{B}\boldsymbol{T} \begin{bmatrix} \bar{V}_+ \\ \bar{V}_- \\ \bar{V}_o \end{bmatrix}_{abc}$$

$$\begin{bmatrix} \bar{I}_+ \\ \bar{I}_- \\ \bar{I}_o \end{bmatrix}_{abc} = \begin{bmatrix} \bar{I}_+ \\ \bar{I}_- \\ \bar{I}_o \end{bmatrix}_{ABC} + j\boldsymbol{T}^{-1}\boldsymbol{B}\boldsymbol{T} \begin{bmatrix} \bar{V}_+ \\ \bar{V}_- \\ \bar{V}_o \end{bmatrix}_{abc}$$

If $\boldsymbol{B}$ has full three-phase symmetry:

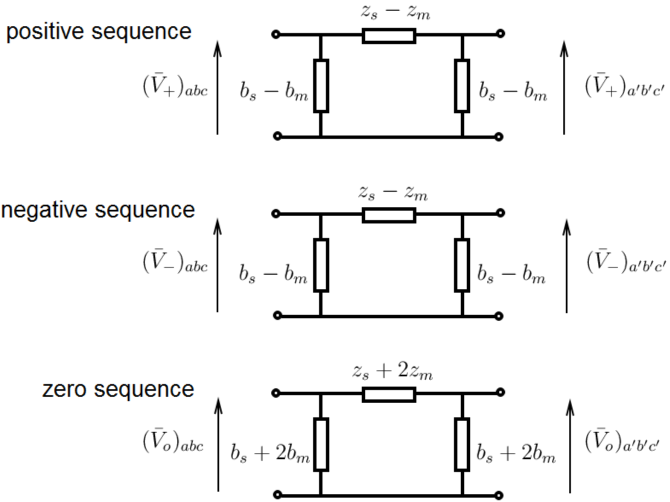
$$\boldsymbol{B} = \begin{bmatrix} b_s & b_m & b_m \\ b_m & b_s & b_m \\ b_m & b_m & b_s \end{bmatrix}$$

then, we have:

$$\boldsymbol{T}^{-1}\boldsymbol{B}\boldsymbol{T} = \begin{bmatrix} b_s - b_m & 0 & 0 \\ 0 & b_s - b_m & 0 \\ 0 & 0 & b_s + 2b_m \end{bmatrix}$$

Equivalent circuits

Under the above mentioned three-phase symmetries, the positive, negative and zero-sequence equivalent circuits are decoupled.
They have the following structure and parameters:



Interpretation of impedances and admittances of the equivalent circuits

- $z_s - z_m$ is the series impedance seen in one phase when positive-sequence currents flow in the line
- $b_s - b_m$ is the half shunt susceptance seen in one phase when the line is subject to positive-sequence voltages
- the positive-sequence equivalent circuit is the per-phase one derived in course ELEC0014. $z_s - z_m$ is the *cyclic impedance*.

- the negative- and positive-sequence parameters of a line are identical since changing the sequence of currents (from positive to negative) does not modify the behaviour of the line.

- $z_s + 2z_m$ is the series impedance seen in one phase when zero-sequence currents flow in the line
- the lines of magnetic field created by such currents are very different from those created by positive-sequence currents. This leads to a zero-sequence reactance 2 to 3.5 times larger than the positive-sequence reactance.

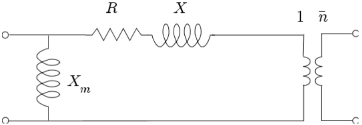
- $b_s + 2b_m$ is the half shunt susceptance seen in one phase when the line is subject to zero-sequence voltages

- the zero-sequence impedance $z_s + 2z_m$ involves the parameters Z_{an} and Z_{nn} relative to the ground and the shield wires. These parameters are not involved in the positive and negative-sequence impedances.
- in practice, the parameters relative to the ground are not known accurately
  - for instance the resistivity of the ground along the line is not well known
- hence, the zero-sequence impedance should be *measured* instead of calculated
- such measurements allow better knowing the positive-sequence parameters also.

Positive, negative and zero-sequence equivalent circuits of a transformer

Positive-sequence equivalent circuit

- The positive-sequence equivalent circuit of the transformer is the per-phase equivalent derived in course ELEC0014:

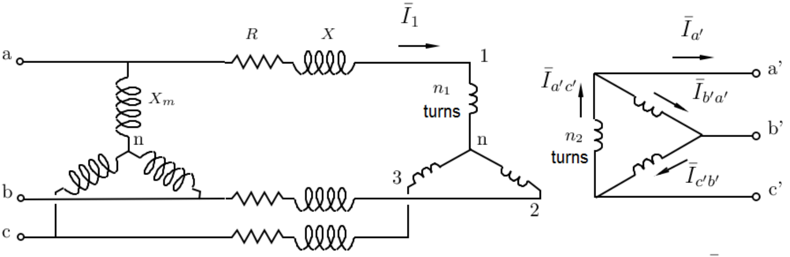


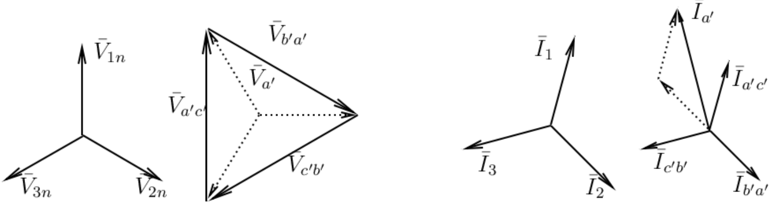
- $\angle n = p\dfrac{\pi}{6}$ where $p \in \{0, 1, 2, \ldots, 11\}$: see IEC code of the vector group

Negative-sequence equivalent circuit

- The negative-sequence impedances are equal to the positive-sequence impedances since changing the phasor sequence does not change the transformer behaviour
- however, the complex transformer ratio $\bar{n}$ is replaced by its conjugate $\bar{n}^\star$ as shown in the following two slides for a transformer Yd11

Yd11 transfo with positive-sequence voltages and currents (see course ELEC0014)

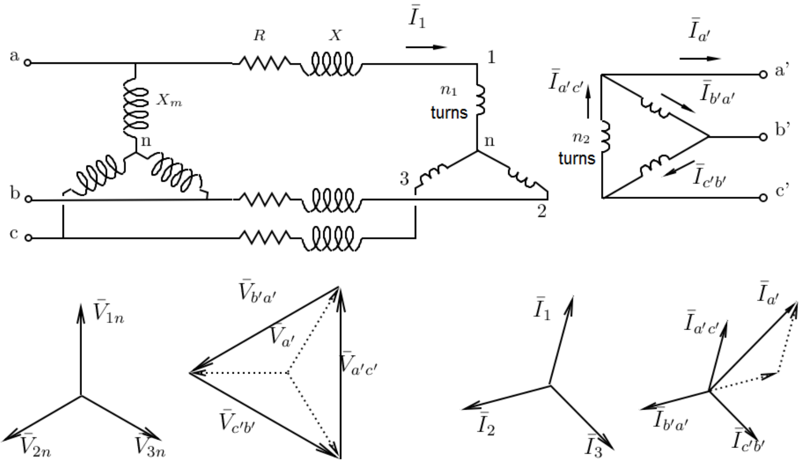




$$\bar{V}_{a'} = \frac{1}{\sqrt{3}}\, e^{j\pi/6}\, \bar{V}_{a'c'} = \frac{n_2}{\sqrt{3} n_1}\, e^{j\pi/6}\, \bar{V}_{1n} = \bar{n} \bar{V}_{1n} \qquad \text{with} \qquad \bar{n} = \frac{n_2}{\sqrt{3}\, n_1}\, e^{j\pi/6}$$

$$\bar{I}_{a'} = \sqrt{3}\, e^{j\pi/6}\, \bar{I}_{a'c'} = \frac{\sqrt{3}\, n_1}{n_2}\, \frac{1}{e^{-j\pi/6}}\, \bar{I}_1 = \frac{1}{\bar{n}^\star}\, \bar{I}_1$$

Same Yd11 transformer with negative-sequence voltages and currents



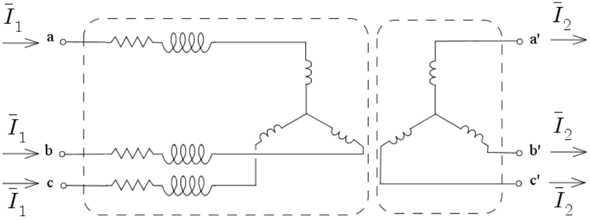
$$\bar{V}_{a'} = \frac{1}{\sqrt{3}}\, e^{-j\pi/6}\, \bar{V}_{a'c'} = \frac{n_2}{\sqrt{3} n_1}\, e^{-j\pi/6}\, \bar{V}_{1n} = \bar{n}^{\star}\, \bar{V}_{1n}$$

$$\bar{I}_{a'} = \sqrt{3}\, e^{-j\pi/6}\, \bar{I}_{a'c'} = \frac{\sqrt{3}\, n_1}{n_2}\, \frac{1}{e^{j\pi/6}}\, \bar{I}_1 = \frac{1}{\bar{n}}\, \bar{I}_1$$

Zero-sequence equivalent circuit

It what follows the magnetizing reactance X_m has been assumed infinite, to make the figures more legible, but there is no problem adding it for better accuracy.

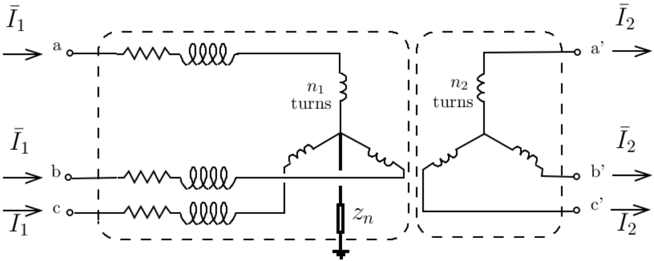
Yy0 transformer



Kirchhoff current law: $3\bar{I}_1 = 0 \quad \Rightarrow \quad \bar{I}_1 = 0$
Kirchhoff current law: $3\bar{I}_2 = 0 \quad \Rightarrow \quad \bar{I}_2 = 0$

Hence, the zero-sequence equivalent is:

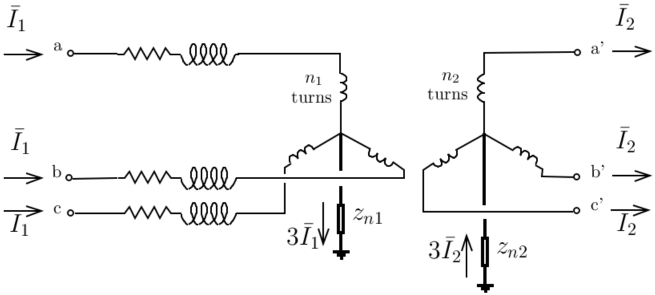
Yny0 transformer



Kirchhoff current law: $3\bar{I}_2 = 0 \quad \Rightarrow \quad \bar{I}_2 = 0$
Ideal transformer relation: $\bar{I}_1 = \dfrac{n_2}{n_1}\bar{I}_2 \quad \Rightarrow \quad \bar{I}_1 = 0$

Hence, the zero-sequence equivalent is:

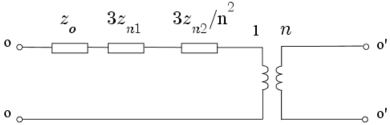
Ynyn0 transformer



The three currents $\bar{I}_1$ can circulate thanks to the neutral grounding

The same holds true for $\bar{I}_2$, with $\bar{I}_2 = \dfrac{n_1}{n_2}\bar{I}_1$

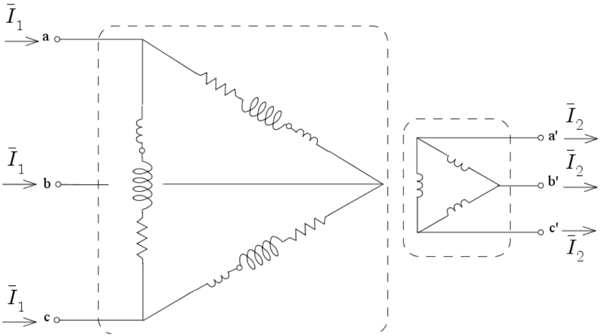
Hence, the zero-sequence equivalent is:



$n = n_2/n_1$

z_o : zero-sequence impedance of the transformer itself (see slide # 41)

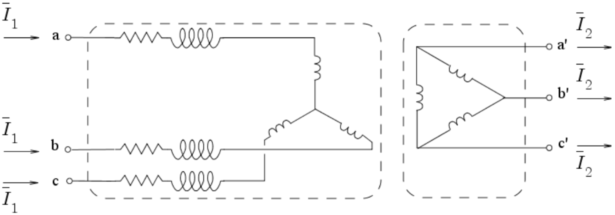
Dd0 transformer



Kirchhoff current law: $3\bar{I}_1 = 0 \quad \Rightarrow \quad \bar{I}_1 = 0$
Kirchhoff current law: $3\bar{I}_2 = 0 \quad \Rightarrow \quad \bar{I}_2 = 0$

Hence, the zero-sequence equivalent is:

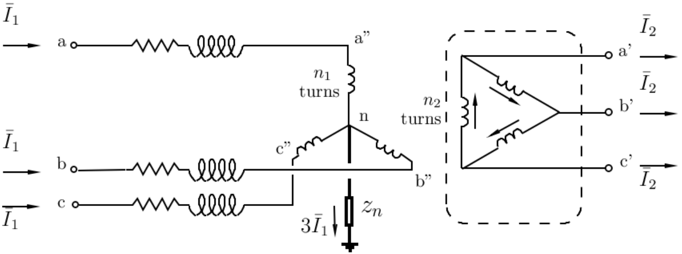
Yd* transformateur



Kirchhoff current law: $3\overline{I}_1 = 0 \quad \Rightarrow \quad \overline{I}_1 = 0$
Kirchhoff current law: $3\overline{I}_2 = 0 \quad \Rightarrow \quad \overline{I}_2 = 0$

Hence, the zero-sequence equivalent is:

Ynd* transformer



Kirchhoff current law: $3\bar{I}_2 = 0 \quad \Rightarrow \quad \bar{I}_2 = 0$

The three currents $\bar{I}_1$ can circulate thanks to the neutral grounding and because the current $\bar{I}_1 \dfrac{n_1}{n_2}$ can circulate in the triangle.

Under the effect of zero-sequence currents:

$$\bar{V}_{a''} - \bar{V}_n = \bar{V}_{b''} - \bar{V}_n = \bar{V}_{c''} - \bar{V}_n$$

Hence, on the secondary of the ideal transformers:

$$\bar{V}_{a'} - \bar{V}_{c'} = \bar{V}_{b'} - \bar{V}_{a'} = \bar{V}_{c'} - \bar{V}_{b'} \tag{1}$$

Kirchhoff voltage law gives:

$$(\bar{V}_{a'} - \bar{V}_{c'}) + (\bar{V}_{b'} - \bar{V}_{a'}) + (\bar{V}_{c'} - \bar{V}_{b'}) = 0$$

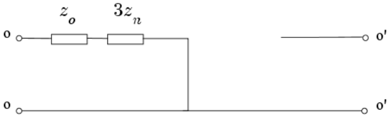
Combining this with (1):

$$\bar{V}_{a'} - \bar{V}_{c'} = \bar{V}_{b'} - \bar{V}_{a'} = \bar{V}_{c'} - \bar{V}_{b'} = 0$$

and coming back to the primary of the ideal transformers:

$$\bar{V}_{a''} - \bar{V}_{n} = \bar{V}_{b''} - \bar{V}_{n} = \bar{V}_{c''} - \bar{V}_{n} = 0$$

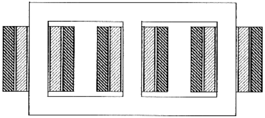
Hence, the zero-sequence equivalent is:



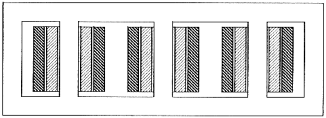
z_o : zero-sequence impedance of the transformer itself (see next slide)

Zero-sequence impedance

1. The three phases are on separate cores
   - each transformer carries one of the zero-sequence currents, and "is not aware" of the sequence of applied voltages
   - the zero-sequence equivalent circuit is identical to the positive-sequence one.
2. The three phases are mounted on a common iron core
   - the leakage reactance is (almost) the same, whatever the sequence of currents
   - the magnetizing susceptance is different, depending on the configuration:



three-leg configuration



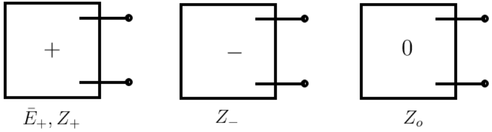
five-leg configuration

   - the magnetic fields created by the zero-sequence currents oppose to each other more or less strongly inside the magnetic core
   - this leads to a small magnetic flux in each phase of the three-leg configuration
   - which translates into a magnetizing reactance much smaller in the zero-sequence equivalent circuit than in the positive-sequence one.

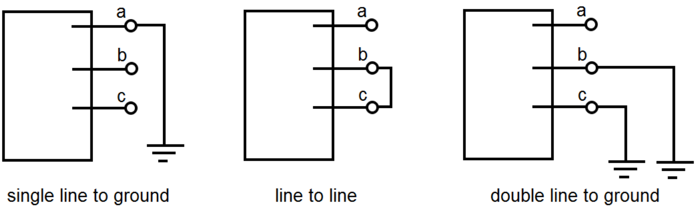
Assembling the sequence networks according to the fault

The equivalent circuits of the various lines, cables, transformers, generators and loads can be assembled into one positive-sequence, one negative-sequence and one zero-sequence network, respectively.

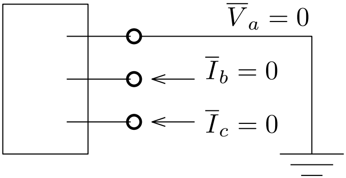
Consider these equivalent networks seen from a given bus.

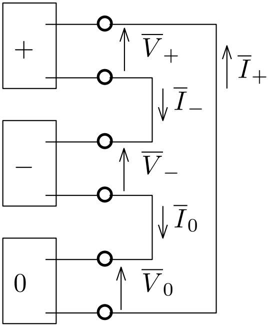


Connect these networks to account for each of the faults shown below, taking place at the bus of concern.



Single-line-to-ground fault

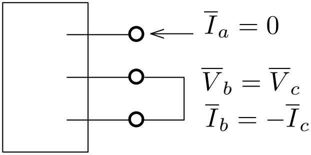


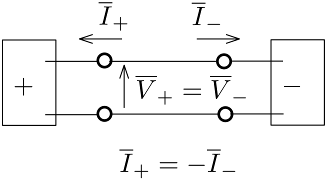


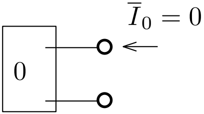
$$\overline{V}_+ + \overline{V}_- + \overline{V}_0 = 0$$

$$\overline{I}_+ = \overline{I}_- = \overline{I}_0$$

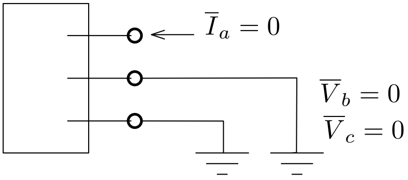
Line-to-line fault

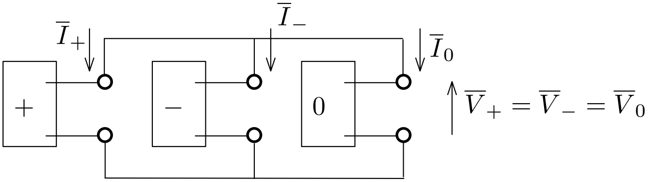






Double-line-to-ground fault





$$\overline{I}_+ + \overline{I}_- + \overline{I}_0 = 0$$