ELEC0029 - Electric Power system analysis

Behaviour of synchronous machine during a short-circuit (a simple example of electromagnetic transients)

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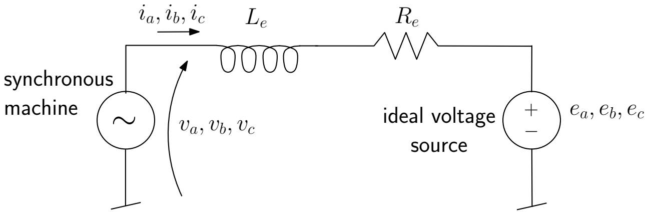
System modelling



Network represented by a simple Thévenin equivalent :

- resistance R_e and inductance L_e in each phase
- no magnetic coupling between phases, for simplicity

Machine :

- only the field winding f in the d axis
- only one damper winding $q1$ in the q axis
- rotor speed $\dot{\theta}_r$ assumed constant
  - the focus is on short-lasting electromagnetic transients
- constant excitation voltage V_f
  - it is assumed that the automatic voltage regulator hasn't time to react

Network equations

$$v_a - e_a = R_e i_a + L_e \frac{di_a}{dt} \qquad \text{with} \qquad e_a = \sqrt{2}E\cos(\omega_N t + \theta) \quad (1)$$

$$v_b - e_b = R_e i_b + L_e \frac{di_b}{dt} \qquad \text{with} \qquad e_b = \sqrt{2}E\cos(\omega_N t + \theta - \frac{2\pi}{3}) \quad (2)$$

$$v_c - e_c = R_e i_c + L_e \frac{di_c}{dt} \qquad \text{with} \qquad e_c = \sqrt{2}E\cos(\omega_N t + \theta - \frac{4\pi}{3}) \quad (3)$$

Park transformation

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = \mathcal{P} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = \mathcal{P} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (4)$$

$$\text{with } \mathcal{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{4\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{4\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (5)$$

where $\theta_r = \theta_r^o + \omega_N t$

Machine equations

$$\psi_d = L_{dd} i_d + L_{df} i_f \quad (6)$$

$$\psi_q = L_{qq} i_q + L_{qq1} i_{q1} \quad (7)$$

$$\psi_o = L_{oo} i_o \quad (8)$$

$$\psi_f = L_{ff} i_f + L_{df} i_d \quad (9)$$

$$\psi_{q1} = L_{q1q1} i_{q1} + L_{qq1} i_q \quad (10)$$

$$v_d = -R_a i_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt} \quad (11)$$

$$v_q = -R_a i_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt} \quad (12)$$

$$v_o = -R_a i_o - \frac{d\psi_o}{dt} \quad (13)$$

$$V_f = R_f i_f + \frac{d\psi_f}{dt} \quad (14)$$

$$0 = R_{q1} i_{q1} + \frac{d\psi_{q1}}{dt} \quad (15)$$

Variable - equations balance

- 19 variables : $v_a, v_b, v_c, i_a, i_b, i_c, v_d, v_q, v_o, i_d, i_q, i_o, \psi_d, \psi_q, \psi_o, \psi_f, \psi_{q1}, i_f, i_{q1}$
- 19 equations : (1 - 3), 6 eqs. in (4), (6 - 15)

Remarks

- The model is made up of Differential-Algebraic Equations (DAEs)
- some of the variables and some of the equations could be eliminated but the additional computational effort of keeping all of them is negligible[1]
- θ_r being known, the equations are linear with respect to the unknowns
- some coefficients in these equations vary with time.

[1] not to mention the risk of introducing mistakes in analytical manipulations !

Passing the equations in per unit

At the stator (a, b, c):

- base voltage V_B = nominal RMS phase-to-neutral voltage (kV)
- base power S_B = single-phase apparent power (MVA)
- base current $I_B = S_B / V_B$, base magnetic flux $\psi_B = V_B / \omega_N$, etc.

In each of the Park winding (d, q, o):

- base voltage $= V_B$
- base power $= S_B$
- base current $= S_B / V_B = I_B$, base flux $= V_B / \omega_N$, etc.

The rotor variables and equations are also transformed in pu; the system is not detailed here.

After passing in per unit:

- $\dot{\theta}_r = 1$ pu in Eqs. (11, 12)
- each time derivative is multiplied by $1/\omega_N$, since we keep the time t in seconds (not in pu)

Equations converted in per unit and rearranged

$$\frac{1}{\omega_N}\frac{di_a}{dt} = -\frac{R_e}{L_e}i_a + \frac{1}{L_e}v_a - \frac{1}{L_e}e_a \quad (16)$$

$$\frac{1}{\omega_N}\frac{di_b}{dt} = -\frac{R_e}{L_e}i_b + \frac{1}{L_e}v_b - \frac{1}{L_e}e_b \quad (17)$$

$$\frac{1}{\omega_N}\frac{di_c}{dt} = -\frac{R_e}{L_e}i_c + \frac{1}{L_e}v_c - \frac{1}{L_e}e_c \quad (18)$$

$$0 = \sqrt{\frac{2}{3}}\left[\cos(\theta_r)v_a + \cos(\theta_r - \frac{2\pi}{3})v_b + \cos(\theta - \frac{4\pi}{3})v_c\right] - v_d \quad (19)$$

$$0 = \sqrt{\frac{2}{3}}\left[\sin(\theta_r)v_a + \sin(\theta_r - \frac{2\pi}{3})v_b + \sin(\theta_r - \frac{4\pi}{3})v_c\right] - v_q \quad (20)$$

$$0 = \frac{1}{\sqrt{3}}(v_a + v_b + v_c) - v_o \quad (21)$$

$$0 = \sqrt{\frac{2}{3}}\left[\cos(\theta_r)i_a + \cos(\theta_r - \frac{2\pi}{3})i_b + \cos(\theta_r - \frac{4\pi}{3})i_c\right] - i_d \quad (22)$$

$$0 = \sqrt{\frac{2}{3}}\left[\sin(\theta_r)i_a + \sin(\theta_r - \frac{2\pi}{3})i_b + \sin(\theta_r - \frac{4\pi}{3})i_c\right] - i_q \quad (23)$$

$$0 = \frac{1}{\sqrt{3}}(i_a + i_b + i_c) - i_o \quad (24)$$

$$
\begin{aligned}
0 &= L_{dd} i_d + L_{df} i_f - \psi_d && (25)\\
0 &= L_{qq} i_q + L_{qq1} i_{q1} - \psi_q && (26)\\
0 &= L_{ff} i_f + L_{df} i_d - \psi_f && (27)\\
0 &= L_{q1q1} i_{q1} + L_{qq1} i_q - \psi_{q1} && (28)\\
0 &= L_{oo} i_o - \psi_o && (29)\\
\frac{1}{\omega_N}\frac{d\psi_d}{dt} &= -R_a i_d - \psi_q - v_d && (30)\\
\frac{1}{\omega_N}\frac{d\psi_q}{dt} &= -R_a i_q + \psi_d - v_q && (31)\\
\frac{1}{\omega_N}\frac{d\psi_f}{dt} &= -R_f i_f + V_f && (32)\\
\frac{1}{\omega_N}\frac{d\psi_{q1}}{dt} &= -R_{q1} i_{q1} && (33)\\
\frac{1}{\omega_N}\frac{d\psi_o}{dt} &= -R_a i_o - v_o && (34)
\end{aligned}
$$

Model in compact form

With a proper reordering of equations and states, the model can be rewritten in compact form as:

$$(1/\omega_N)\,\dot{\boldsymbol{x}} = \boldsymbol{A}_{xx}\boldsymbol{x} + \boldsymbol{A}_{xy}\boldsymbol{y} + \boldsymbol{u}_x \quad (35)$$

$$\boldsymbol{0} = \boldsymbol{A}_{yx}\boldsymbol{x} + \boldsymbol{A}_{yy}\boldsymbol{y} + \boldsymbol{u}_y \quad (36)$$

where:

$$\begin{aligned}
\boldsymbol{x} &= [\, i_a\ i_b\ i_c\ \psi_d\ \psi_q\ \psi_f\ \psi_{q1}\ \psi_o \,]^T \\
\boldsymbol{y} &= [\, v_a\ v_b\ v_c\ v_d\ v_q\ v_o\ i_d\ i_q\ i_o\ i_f\ i_{q1} \,]^T \\
\boldsymbol{u}_x &= [\, -\frac{e_a}{L_e}\ -\frac{e_b}{L_e}\ -\frac{e_c}{L_e}\ 0\ 0\ V_f\ 0\ 0 \,]^T \\
\boldsymbol{u}_y &= [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T
\end{aligned}$$

Numerical solution of the DAEs

Let k denote the discrete time ($k = 0, 1, 2, \ldots$), and h the time step size.

A popular numerical integration formula is the *Trapezoidal Method* :

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \frac{h}{2}\left(\dot{\boldsymbol{x}}_{k+1} + \dot{\boldsymbol{x}}_k\right)$$

Replacing $\dot{\boldsymbol{x}}_{k+1}$ by its expression (35) :

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \frac{h}{2}\omega_N \boldsymbol{A}_{xx}\boldsymbol{x}_{k+1} + \frac{h}{2}\omega_N \boldsymbol{A}_{xy}\boldsymbol{y}_{k+1} + \frac{h}{2}\omega_N\, \boldsymbol{u}_{x\ k+1} + \frac{h}{2}\dot{\boldsymbol{x}}_k$$

Dividing by $\dfrac{h\omega_N}{2}$ and rearranging the various terms :

$$\left[\boldsymbol{A}_{xx} - \frac{2}{h\omega_N}\boldsymbol{I}\right]\boldsymbol{x}_{k+1} + \boldsymbol{A}_{xy}\boldsymbol{y}_{k+1} = -\frac{2}{h\omega_N}\boldsymbol{x}_k - \frac{1}{\omega_N}\dot{\boldsymbol{x}}_k - \boldsymbol{u}_{x\ k+1} \tag{37}$$

where $\boldsymbol{I}$ is the unit matrix of same dimension as $\boldsymbol{x}$.

On the other hand, from Eq. (36) we have :

$$\boldsymbol{A}_{yx}\boldsymbol{x}_{k+1} + \boldsymbol{A}_{yy}\boldsymbol{y}_{k+1} = -\boldsymbol{u}_{y\ k+1} \tag{38}$$

Grouping Eqs. (37) and (38), the linear system to solve at each time step is :

$$\begin{bmatrix} \boldsymbol{A}_{xx} - \dfrac{2}{h\omega_N}\boldsymbol{I} & \boldsymbol{A}_{xy} \\ \boldsymbol{A}_{yx} & \boldsymbol{A}_{yy} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k+1} \\ \boldsymbol{y}_{k+1} \end{bmatrix} = \begin{bmatrix} -\dfrac{2}{h\omega_N}\boldsymbol{x}_k - \dfrac{1}{\omega_N}\dot{\boldsymbol{x}}_k - \boldsymbol{u}_{x\ k+1} \\ -\boldsymbol{u}_{y\ k+1} \end{bmatrix} \tag{39}$$

Numerical example and comments on the results

Network and machine data

$f_N = 50$ Hz		
$L_e = 0.20$ pu	$R_e = 0.01$ pu	
$R_a = 0.005$ pu		
$L_{dd} = 2.4$ pu	$L_{df} = 2.2$ pu	$L_{ff} = 2.42$ pu
$L_{qq} = 2.4$ pu	$L_{qq1} = 2.2$ pu	$L_{q1q1} = 2.2512$ pu
$R_f = 0.0011$ pu	$R_{q1} = 0.0239$ pu	
$L_{oo} = 0.1$ pu		

Initial operating point

$P = 0.5$ pu
$Q = 0.1$ pu
$\bar{V}_a = 1.000 \text{ pu } \angle 0$

Simulation results

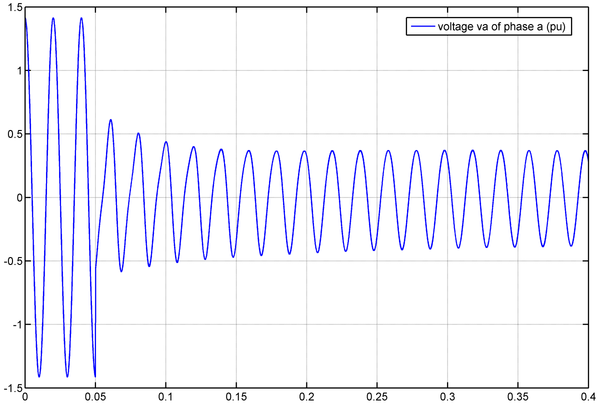
A three-phase short-circuit is simulated by setting E to zero at $t = 0.05$ s.

Important remark

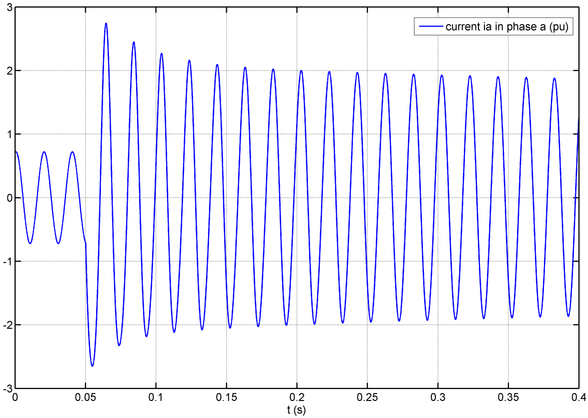
The fault is *not* cleared in order to show the various time constants present in the current evolution.

However, in practice:

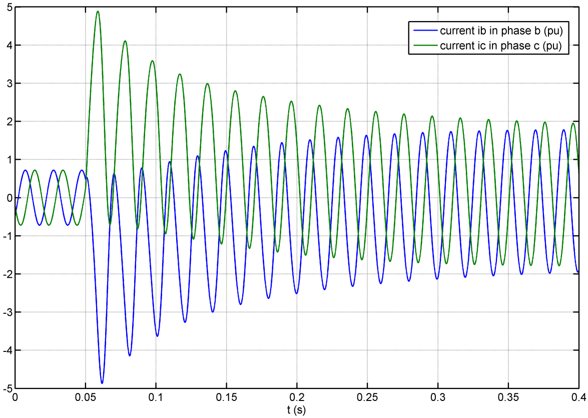
- the fault must be cleared fast enough, e.g. after 5 - 10 cycles (0.1 - 0.2 s)
- beyond that time, the model is no longer valid:
  - rotor speed would not remain constant
  - v_f would be adjusted by the Automatic Voltage Regulator
  - etc.



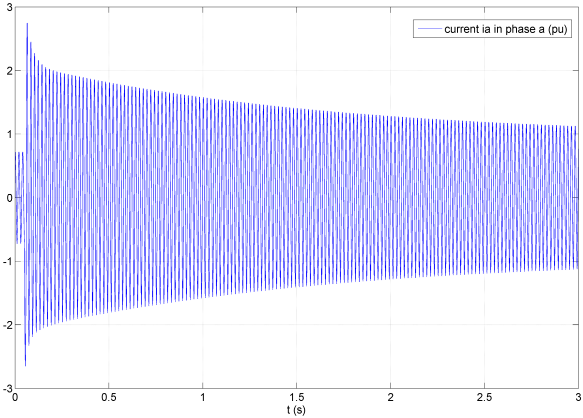
- decrease of AC voltage magnitude under the effect of the fault
- presence of a residual voltage due to some emf inside the generator
- remark:
  - such an emf does not exist in generators connected to the network through power electronic interfaces (dispersed generation in MV distribution grid)
  - the latter do not participate to the short-circuit capacity!



- increase of amplitude of alternating current under the effect of the fault
- the envelop of the current wave varies with time (more details in the sequel)
- presence of a small *aperiodic* or *unidirectional* or *DC* component
  - typical of transients in an RL circuit due to switchings
  - much more visible in the other two phases: see next slide



- the magnitude of the aperiodic components decrease with a time constant $\simeq 0.15$ s (in this example)
- the aperiodic components are not the same in all three phases, because the rotor is not in the same position with respect to each stator winding
- once they have vanished, the three phase currents become again sinusoidal and balanced



- the magnitude of the alternating component of i_a shows two time constants:
  - a short one, lasting a few cycles, resulting in a slightly higher initial amplitude of the current: caused by damper winding $q1$
  - a much longer one ($\simeq 1.5$ s in this example): caused by field winding f
- the current that the breakers have to interrupt is much higher than the one which would prevail in steady-state !
- the machine behaves initially as if it had a smaller internal reactance

Magnetic fields

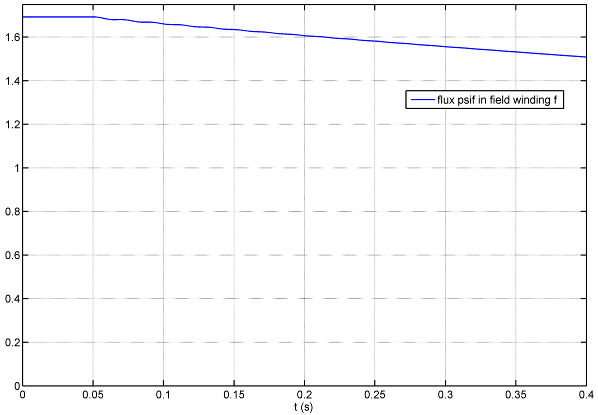
The alternating components of the stator currents i_a, i_b and i_c

- are shifted by $\pm 2\pi/3$ rad. They create a magnetic field H_{AC} which rotates at the angular speed ω_N
- this field is fixed with respect to the rotor windings
- under the effect of the fault, the amplitude if i_a, i_b and i_c increases significantly. So does the magnetic field H_{AC}
- this induces aperiodic current components in the rotor windings.

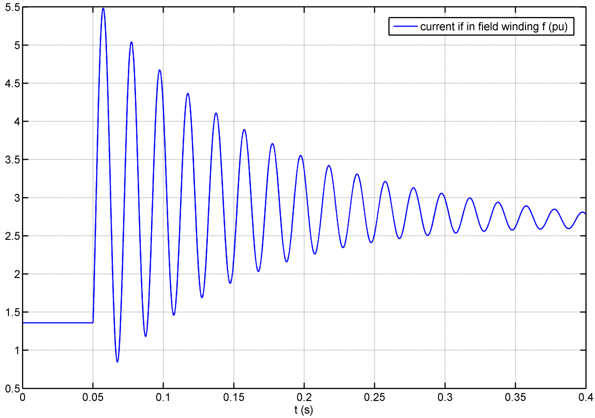
The aperiodic components of the stator currents i_a, i_b and i_c

- create a magnetic field H_{DC} which is fixed with respect to the stator
- hence it rotates at angular speed ω_N with respect to the rotor windings
- this induces alternating components of angular frequency ω_N in the rotor windings.

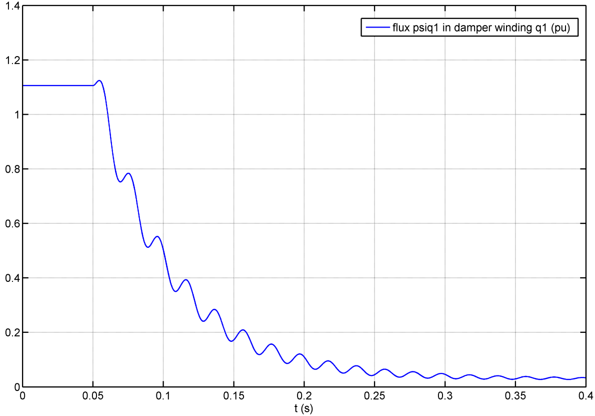
This is confirmed by the plots in the next slides.



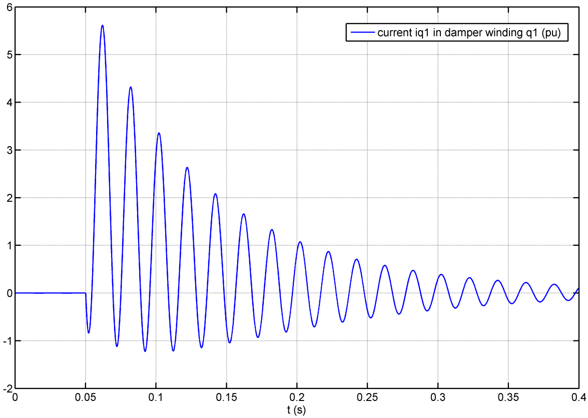
- the flux ψ_f in the field winding changes very little in spite of the large increase of stator currents !
- large "magnetic inertia" due to the long time constant $L_{ff}/R_f (= 7$ s in this example)



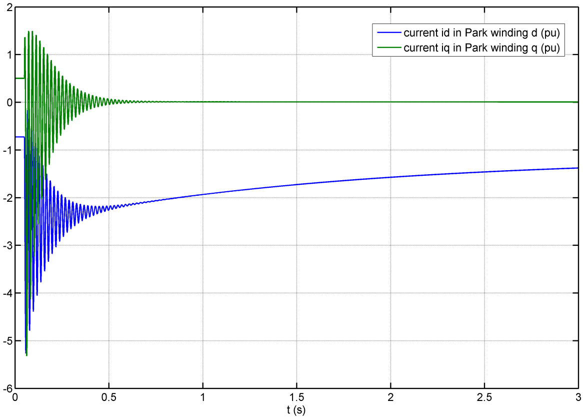
- Lenz law: additional current components appear in the field winding in order to keep ψ_f (almost) constant
- the oscillatory component is due to the magnetic field H_{DC}
  - check: time constant of decay = time constant of aperiodic component of stator currents $\simeq 0.15$ s
- the aperiodic component is due to the magnetic field H_{AC}
  - time constant $\simeq 1.5$ s, observed in slide 17



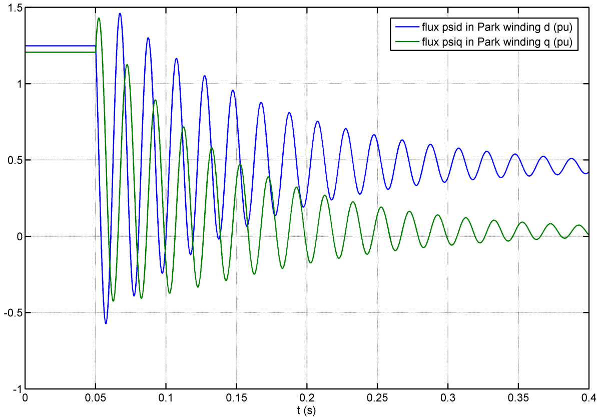
- the flux in the damper winding $q1$ is comparatively more “volatile”
- indeed, the field and the damper windings are constructively very different: field coil vs. damper bars in rotor slots



- the damper current i_{q1} has a zero initial (and final) value
- the oscillatory component is due to the magnetic field H_{DC}
- the aperiodic component decreases much faster than the aperiodic component of i_f
- it corresponds to the initial, fast decaying, increment of the stator current amplitude (see slides 16 and 17)



- the oscillatory component of i_d (resp. i_q) corresponds to the oscillatory component of i_f (resp. i_{q1}) which lies on the same axis
- it can be shown that i_q goes to almost zero due to the predominantly inductive nature of the short-circuit
- the aperiodic component of i_d evolves with the long time constant observed for the aperiodic component of i_f



- fluxes ψ_d and ψ_q in Park windings vary comparatively much faster
- since i_q and i_{q1} tend to zero, so does ψ_q.

Simplified model of a synchronous machine for use in short-circuit calculations

Usual simplifications for the computation of fault currents

- The aperiodic components of the short-circuit currents are neglected
  - by the time the circuit breaker opens, these components are already small
  - this approximation can be compensated by multiplying the fault current by an empirical factor larger than one
- only the alternating components at frequency ω_N are considered.
- this allows using static computations as in sinusoidal steady-state !

Simple representation of the machine

- in steady state, a (round-rotor) machine can be represented by the stator resistance and the synchronous reactance in series with an emf whose magnitude is proportional to the field current
- let us derive a similar model to represent the machine in the first cycles after the fault occurrence.

Park equations of the machine:

$$v_d = -R_a i_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt} = -R_a i_d - \dot{\theta}_r \left(L_{qq} i_q + L_{qq_1} i_{q_1}\right) - \frac{d\psi_d}{dt} \quad (40)$$

$$v_q = -R_a i_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt} = -R_a i_q + \dot{\theta}_r \left(L_{dd} i_d + L_{df} i_f\right) - \frac{d\psi_q}{dt} \quad (41)$$

Just after the short-circuit, the fluxes ψ_f and ψ_{q1} cannot change (much). On the contrary, i_f and i_{q1} vary significantly to ensure constant fluxes.

Let us transform Eqs. (40, 41) to make ψ_f and ψ_{q1} appear.

$$\psi_f = L_{ff} i_f + L_{df} i_d \quad \Rightarrow \quad i_f = \frac{\psi_f - L_{df} i_d}{L_{ff}}$$

$$\psi_{q1} = L_{q1q1} i_{q1} + L_{qq1} i_{q1} \quad \Rightarrow \quad i_{q1} = \frac{\psi_{q1} - L_{qq1} i_q}{L_{q1q1}}$$

Introducing these expressions in Eqs. (40, 41):

$$v_d = -R_a i_d - \dot{\theta}_r \left(L_{qq} - \frac{L_{qq1}^2}{L_{q1q1}}\right) i_q - \dot{\theta}_r \frac{L_{qq1}}{L_{q1q1}} \psi_{q1} - \frac{d\psi_d}{dt} \quad (42)$$

$$v_q = -R_a i_q + \dot{\theta}_r \left(L_{dd} - \frac{L_{df}^2}{L_{ff}}\right) i_d + \dot{\theta}_r \frac{L_{df}}{L_{ff}} \psi_f - \frac{d\psi_q}{dt} \quad (43)$$

Simplifying assumptions

- rotor speed : $\dot{\theta}_r = \omega_N$
- the transformer emf's are neglected : $\frac{d\psi_d}{dt} = 0 \quad \frac{d\psi_q}{dt} = 0$

  This leads to neglecting aperiodic components of short-circuit currents.

Eqs. (42, 43) become:

$$v_d = -R_a i_d - \omega_N \underbrace{(L_{qq} - \frac{L_{qq1}^2}{L_{q1q1}})}_{L_q''} i_q \underbrace{-\omega_N \frac{L_{qq1}}{L_{q1q1}} \psi_{q1}}_{e_d''} = -R_a i_d - X_q'' \, i_q + e_d'' \quad (44)$$

$$v_q = -R_a i_q + \omega_N \underbrace{(L_{dd} - \frac{L_{df}^2}{L_{ff}})}_{L_d'} i_d \underbrace{+\omega_N \frac{L_{df}}{L_{ff}} \psi_f}_{e_q'} = -R_a i_q + X_d' \, i_d + e_q' \quad (45)$$

Compare to the model in steady state:

$$v_d = -R_a i_d - \omega_N L_{qq} i_q = -R_a i_d - X_q i_q \quad (46)$$

$$v_q = -R_a i_q + \omega_N L_{dd} i_d + E_q = -R_a i_q + X_d i_d + E_q \quad (47)$$

The models (44, 45) and (46, 47) are similar except for the following:

- the direct-axis synchronous reactance $X_d = \omega_N L_{dd}$ is replaced by the smaller reactance $X'_d = \omega_N L'_d$
- the quadrature-axis synchronous reactance $X_q = \omega_N L_{qq}$ is replaced by the smaller reactance $X''_q = \omega_N L''_q$
- the emf $\bar{E}_q$, located along the q axis, is replaced by an emf with components in both axes
- for some time after the fault inception, these components keep their pre-fault values (since they are proportional to fluxes).

Terminology

- L'_d (resp. X'_d) is called the *direct-axis transient inductance* (resp. *reactance*)
- it stems from the reaction of the field winding, whose dynamics are in the order of a second
- L''_q (resp. X''_q) is called the *quadrature-axis subtransient inductance* (resp. *reactance*)
- it stems from the reaction of the damper winding, whose dynamics are in the order of a few cycles

Equivalent circuit

For higher accuracy, a damper winding d_1 is also considered in the d axis

- this leads to considering a *direct-axis subtransient inductance* (resp. *reactance*) L''_d (resp. X''_d)
- Eqs. (44, 45) become:

$$v_d = -R_a i_d - X''_q i_q + e''_d \quad (48)$$

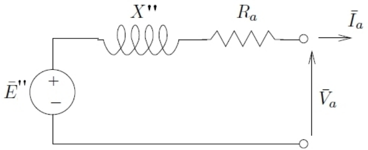
$$v_q = -R_a i_q + X''_d i_d + e''_q \quad (49)$$

- for some time after the fault inception, e''_d and e''_q keep their pre-fault values

In practice $X''_d \simeq X''_q$. Assuming that $X''_d = X''_q = X''$ (*subtransient reactance*), Eqs. (48,49) can be combined into a single complex equation:

$$\bar{V}_a = -R_a \bar{I}_a - jX'' \bar{I}_a + \bar{E}'' \quad (50)$$

which leads to the equivalent circuit:



Typical values

	machine with round rotor (pu)	machine with salient poles (pu)
X_d	1.5 - 2.5	0.9 - 1.5
$X_d^{'}$	0.2 - 0.4	0.3 - 0.5
$X_d^{''}$	0.15 - 0.30	0.25 - 0.35
X_q	1.5 - 2.5	0.5 - 1.1
$X_q^{''}$	0.15 - 0.30	0.25 - 0.35

values in per unit on the machine base power and voltage

Remark

- By considering that the rotor fluxes do not change, the computed current is the one immediately after the fault
- the curve in slide 17 shows that the current magnitude has somewhat decreased by the time the breakers have to act
- we are thus "on the safe side" (little higher current).

How determine the emf $\bar{E}''$?

Let t_{sc} be the instant of the short-circuit inception.

$\bar{E}''$ being the same just after and before the fault:

$$\bar{E}''(t_{sc}^{+}) = \bar{E}''(t_{sc}^{-})$$

while using Eq. (50):

$$\bar{E}''(t_{sc}^{-}) = \bar{V}_a(t_{sc}^{-}) + R_a \bar{I}_a(t_{sc}^{-}) + jX'' \bar{I}_a(t_{sc}^{-})$$

$\bar{V}_a(t_{sc}^{-})$ and $\bar{I}_a(t_{sc}^{-})$ are provided by a power flow computation performed in the pre-fault configuration.

$$P(t_{sc}^{-}) + jQ(t_{sc}^{-}) = \bar{V}_a(t_{sc}^{-})\,\bar{I}_a^{\star}(t_{sc}^{-}) \quad \Rightarrow \quad \bar{I}_a(t_{sc}^{-}) = \frac{P(t_{sc}^{-}) - jQ(t_{sc}^{-})}{\bar{V}_a^{\star}(t_{sc}^{-})}$$