ELEC0029 - Electric Power System Analysis

The synchronous machine (detailed model)

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Objectives

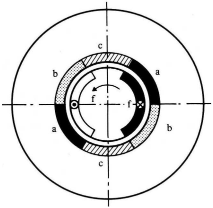
Extend the model of the synchronous machine considered in course ELEC0014:

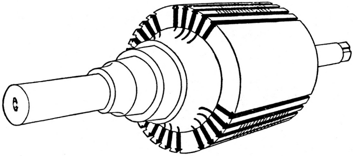
- more detailed
- appropriate for dynamic studies
- includes the effect of damper windings
- applicable to machines with salient-pole rotors (hydro power plants)

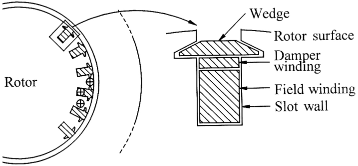
Relies on the Park transformation, also used for other power system components.

The two types of synchronous machines

Round-rotor generators (or turbo-alternators)







- Driven by steam or gas turbines, which rotate at high speed
- one pair of poles (conventional thermal units) or two (nuclear units)
- cylindrical rotor made up of solid steel forging
- diameter << length (centrifugal force !)
- field winding made up of conductors distributed on the rotor, in milled slots
- even if the generator efficiency is around 99 %, the heat produced by Joule losses has to be evacuated !

  Large generators are cooled by hydrogen (heat evacuation 7 times better than air) or water (12 times better) flowing in the hollow stator conductors.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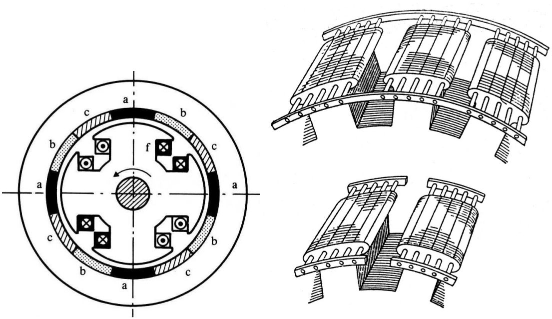
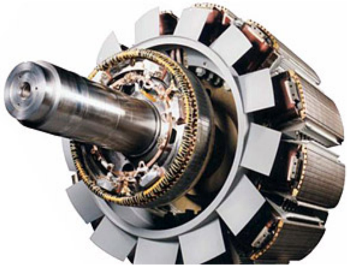
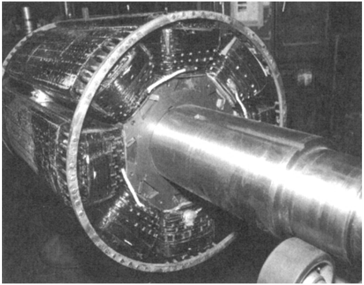
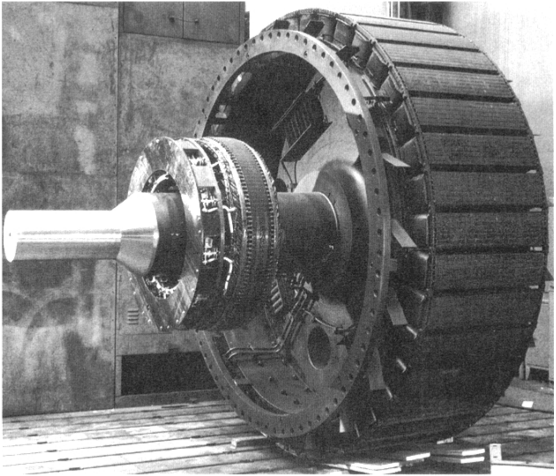
Salient-pole generators



- Driven by hydraulic turbines (or diesel engines), which rotate at low speed
- many pairs of poles (at least 4) $\Rightarrow$ it is more convenient to have field windings concentrated and placed on the poles
- air gap is not constant: min. in front of a pole, max. in between two poles
- poles are shaped to also minimize space harmonics
- diameter $>>$ length (to have space for the many poles)
- rotor is laminated (poles easier to construct)
- generators usually cooled by the flow of air around the rotor.







Damper windings and eddy currents in rotor

Damper windings (or amortisseur)

- round-rotor machines: copper/brass bars placed in the same slots at the field winding, and interconnected to form a damper cage (similar to the squirrel cage of an induction motor)
- salient-pole machines: copper/brass rods embedded in the poles and connected at their ends to rings or segments.

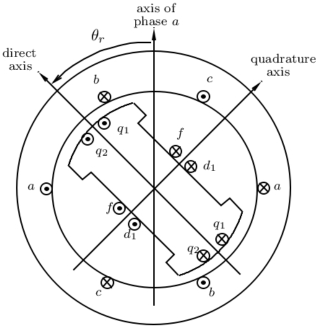
Why?

- in perfect steady state: the magnetic fields produced by both the stator and the rotor are fixed relative to the rotor ⇒ no current induced in dampers
- after a disturbance: the rotor moves with respect to stator magnetic field ⇒ currents are induced in the dampers...
  ...which, according to Lenz's law, create a *damping torque* helping the rotor to align on the stator magnetic field.

Eddy currents in the rotor

Round-rotor generators: the solid rotor offers a path for eddy currents, which produce an effect similar to those of amortisseurs.

Modelling of machine with magnetically coupled circuits



Number of rotor windings = degree of sophistication of model. But:

- more detailed model ⇒ more data are needed
- while measurement devices can be connected only to the field winding.

Most widely used model: 3 or 4 rotor windings:

- f: field winding d_1, q_1: amortisseurs
- q_2: accounts for eddy currents in rotor; not used in (laminated) salient-pole generators.

Remarks

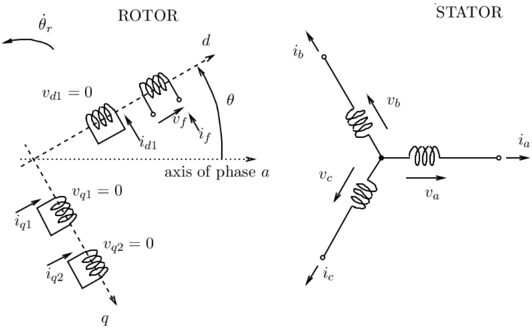
In the sequel, we consider:

- a machine with a single pair of poles, for simplicity.
  This does not affect the electrical behaviour of the generator
  (it affects the moment of inertia and the kinetic energy of rotating masses)

- the general case of a salient-pole machine.
  For a round-rotor machine: set some parameters to the same value in the d and q axes (to account for the equal air gap width)

- the configuration with four rotor windings (f, d_1, q_1, q_2).
  For a salient-pole generator : remove the q_2 winding.

Relations between voltages, currents and magnetic fluxes



Stator windings: generator convention:

$$v_a(t) = -R_a i_a(t) - \frac{d\psi_a}{dt} \qquad v_b(t) = -R_a i_b(t) - \frac{d\psi_b}{dt} \qquad v_c(t) = -R_a i_c(t) - \frac{d\psi_c}{dt}$$

In matrix form:

$$\boldsymbol{v}_T = -\boldsymbol{R}_T \mathbf{i}_T - \frac{d}{dt}\psi_T \quad \text{with} \quad \boldsymbol{R}_T = \text{diag}(R_a \; R_a \; R_a)$$

Rotor windings: motor convention:

$$
\begin{aligned}
v_f(t) &= R_f\, i_f(t) + \frac{d\psi_f}{dt} \\
0 &= R_{d1} i_{d1}(t) + \frac{d\psi_{d1}}{dt} \\
0 &= R_{q1} i_{q1}(t) + \frac{d\psi_{q1}}{dt} \\
0 &= R_{q2} i_{q2}(t) + \frac{d\psi_{q2}}{dt}
\end{aligned}
$$

In matrix form:

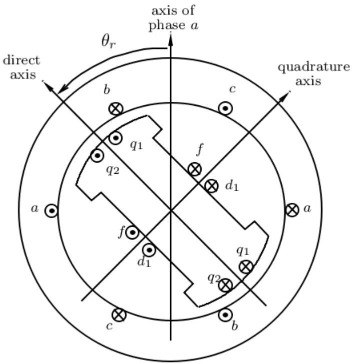
$$
\boldsymbol{v}_r = \boldsymbol{R}_r \mathbf{i}_r + \frac{d}{dt}\boldsymbol{\psi}_r \qquad \text{with} \qquad \boldsymbol{R}_r = \mathrm{diag}(R_f\ R_{d_1}\ R_{q_1}\ R_{q_2})
$$

Inductances

Saturation being neglected, the fluxes vary linearly with the currents according to:

$$\begin{bmatrix} \boldsymbol{\psi}_T \\ \boldsymbol{\psi}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT}(\theta_r) & \boldsymbol{L}_{Tr}(\theta_r) \\ \boldsymbol{L}_{Tr}^T(\theta_r) & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_T \\ \boldsymbol{i}_r \end{bmatrix}$$

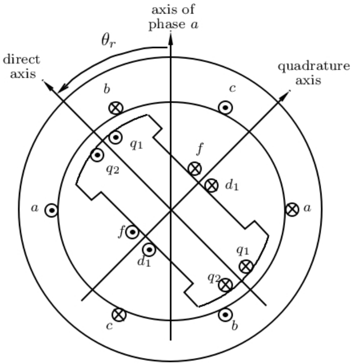
- $\boldsymbol{L}_{TT}$ and $\boldsymbol{L}_{Tr}$ vary with the position θ_r of the rotor
- but $\boldsymbol{L}_{rr}$ does not
- the components of $\boldsymbol{L}_{TT}$ and $\boldsymbol{L}_{Tr}$ are periodic functions of θ_r obviously
- the space harmonics in θ_r are assumed negligible = *sinusoidal machine* assumption.



$$\boldsymbol{L}_{TT}(\theta_r) =$$

$$\begin{bmatrix} L_0 + L_1 \cos 2\theta_r & -L_m - L_1 \cos 2(\theta_r + \frac{\pi}{6}) & -L_m - L_1 \cos 2(\theta_r - \frac{\pi}{6}) \\ -L_m - L_1 \cos 2(\theta_r + \frac{\pi}{6}) & L_0 + L_1 \cos 2(\theta_r - \frac{2\pi}{3}) & -L_m - L_1 \cos 2(\theta_r + \frac{\pi}{2}) \\ -L_m - L_1 \cos 2(\theta_r - \frac{\pi}{6}) & -L_m - L_1 \cos 2(\theta_r + \frac{\pi}{2}) & L_0 + L_1 \cos 2(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

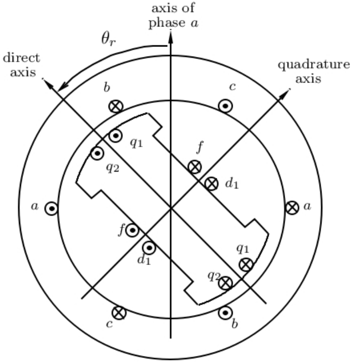
$$L_o, L_1, L_m > 0$$



$$\boldsymbol{L}_{Tr}(\theta_r) =$$

$$\begin{bmatrix} L_{af}\cos\theta_r & L_{ad1}\cos\theta_r & L_{aq1}\sin\theta_r & L_{aq2}\sin\theta_r \\ L_{af}\cos(\theta_r-\frac{2\pi}{3}) & L_{ad1}\cos(\theta_r-\frac{2\pi}{3}) & L_{aq1}\sin(\theta_r-\frac{2\pi}{3}) & L_{aq2}\sin(\theta_r-\frac{2\pi}{3}) \\ L_{af}\cos(\theta_r+\frac{2\pi}{3}) & L_{ad1}\cos(\theta_r+\frac{2\pi}{3}) & L_{aq1}\sin(\theta_r+\frac{2\pi}{3}) & L_{aq2}\sin(\theta_r+\frac{2\pi}{3}) \end{bmatrix}$$

$$L_{af}, L_{ad1}, L_{aq1}, L_{aq2} > 0$$



$$\mathbf{L}_{rr} = \begin{bmatrix} L_{ff} & L_{fd1} & 0 & 0 \\ L_{fd1} & L_{d1d1} & 0 & 0 \\ 0 & 0 & L_{q1q1} & L_{q1q2} \\ 0 & 0 & L_{q1q2} & L_{q2q2} \end{bmatrix}$$

Park transformation and equations

Park transformation

is applied to stator variables (denoted $._T$) to obtain the corresponding Park variables (denoted $._P$):

$$\begin{aligned}
\boldsymbol{v}_P &= \mathcal{P}\ \boldsymbol{v}_T \\
\boldsymbol{\psi}_P &= \mathcal{P}\ \boldsymbol{\psi}_T \\
\boldsymbol{i}_P &= \mathcal{P}\ \boldsymbol{i}_T \\
\text{where} \quad \mathcal{P} &= \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{v}_P &= \begin{bmatrix} v_d & v_q & v_o \end{bmatrix}^T \\
\boldsymbol{\psi}_P &= \begin{bmatrix} \psi_d & \psi_q & \psi_o \end{bmatrix}^T \\
\boldsymbol{i}_P &= \begin{bmatrix} i_d & i_q & i_o \end{bmatrix}^T
\end{aligned}$$

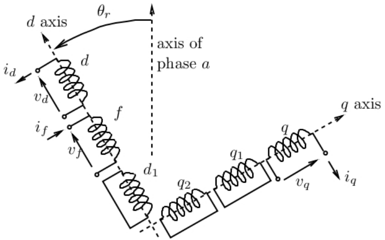
It is easily shown that: $\mathcal{P}\ \mathcal{P}^T = \boldsymbol{I} \quad \Leftrightarrow \quad \mathcal{P}^{-1} = \mathcal{P}^T$

Interpretation

Total magnetic field created by the currents i_a, i_b et i_c:

$$\text{projected on } d \text{ axis:} \quad k\,(\cos\theta_r\, i_a + \cos(\theta_r - \frac{2\pi}{3})\, i_b + \cos(\theta_r - \frac{4\pi}{3})\, i_c) \;=\; k\sqrt{\frac{3}{2}}\, i_d$$

$$\text{projected on } q \text{ axis:} \quad k\,(\sin\theta_r\, i_a + \sin(\theta_r - \frac{2\pi}{3})\, i_b + \sin(\theta_r - \frac{4\pi}{3})\, i_c) \;=\; k\sqrt{\frac{3}{2}}\, i_q$$



The Park transformation consists of replacing the (a, b, c) stator windings by three equivalent windings (d, q, o):

- the d winding is attached to the d axis
- the q winding is attached to the q axis
- the currents i_d and i_q produce together the same magnetic field, to the multiplicative constant $\sqrt{\frac{3}{2}}$.

Park equations of the synchronous machine

$$\begin{aligned}
\boldsymbol{v}_T &= -\boldsymbol{R}_T \boldsymbol{i}_T - \frac{d}{dt}\boldsymbol{\psi}_T \\
\mathcal{P}^{-1}\, \boldsymbol{v}_P &= -R_a \boldsymbol{I}\, \mathcal{P}^{-1}\, \boldsymbol{i}_P - \frac{d}{dt}(\mathcal{P}^{-1}\, \boldsymbol{\psi}_P) \\
\boldsymbol{v}_P &= -R_a \mathcal{P}\mathcal{P}^{-1} \boldsymbol{i}_P - \mathcal{P}\,(\frac{d}{dt}\mathcal{P}^{-1})\boldsymbol{\psi}_P - \mathcal{P}\mathcal{P}^{-1}\frac{d}{dt}\boldsymbol{\psi}_P \\
&= -\boldsymbol{R}_P\, \boldsymbol{i}_P - \dot{\theta}_r \boldsymbol{P}\boldsymbol{\psi}_P - \frac{d}{dt}\boldsymbol{\psi}_P
\end{aligned}$$

with: $\boldsymbol{R}_P = \boldsymbol{R}_T \quad \boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

By decomposing:

$$\begin{aligned}
v_d &= -R_a i_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt} \\
v_q &= -R_a i_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt} \\
v_o &= -R_a i_o - \frac{d\psi_o}{dt}
\end{aligned}$$

$\dot{\theta}_r \psi_d, \dot{\theta}_r \psi_q$: *speed voltages* $\qquad$ $d\psi_d/dt, d\psi_q/dt$: *transformer voltages*

Park inductance matrix

$$\begin{bmatrix} \psi_T \\ \psi_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT} & \boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_T \\ \boldsymbol{i}_r \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{P}^{-1}\psi_P \\ \psi_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{TT} & \boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \mathcal{P}^{-1}\boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix}$$

$$\begin{bmatrix} \psi_P \\ \psi_r \end{bmatrix} = \begin{bmatrix} \mathcal{P}\boldsymbol{L}_{TT}\mathcal{P}^{-1} & \mathcal{P}\boldsymbol{L}_{Tr} \\ \boldsymbol{L}_{Tr}^T\mathcal{P}^{-1} & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{PP} & \boldsymbol{L}_{Pr} \\ \boldsymbol{L}_{rP} & \boldsymbol{L}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_P \\ \boldsymbol{i}_r \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{L}_{PP} & \boldsymbol{L}_{Pr} \\ \boldsymbol{L}_{rP} & \boldsymbol{L}_{rr} \end{bmatrix} = \begin{bmatrix} L_{dd} & & & L_{df} & L_{dd1} & & \\ & L_{qq} & & & & L_{qq1} & L_{qq2} \\ & & L_{oo} & & & & \\ L_{df} & & & L_{ff} & L_{fd1} & & \\ L_{dd1} & & & L_{fd1} & L_{d1d1} & & \\ & L_{qq1} & & & & L_{q1q1} & L_{q1q2} \\ & L_{qq2} & & & & L_{q1q2} & L_{q2q2} \end{bmatrix}$$

(zero entries have been left empty for legibility) with:

$$
\begin{aligned}
L_{dd} &= L_0 + L_m + \frac{3}{2} L_1 \\
L_{qq} &= L_0 + L_m - \frac{3}{2} L_1 \\
L_{df} &= \sqrt{\frac{3}{2}} L_{af} \\
L_{dd1} &= \sqrt{\frac{3}{2}} L_{ad1} \\
L_{qq1} &= \sqrt{\frac{3}{2}} L_{aq1} \\
L_{qq2} &= \sqrt{\frac{3}{2}} L_{aq2} \\
L_{oo} &= L_0 - 2L_m
\end{aligned}
$$

- All components are independent of the rotor position θ_r. That was expected !
- There is no magnetic coupling between d and q axes
  (this was already assumed in $\boldsymbol{L}_{Tr}$ and $\boldsymbol{L}_{rr}$: zero mutual inductances between coils with orthogonal axes).

Leaving aside the o component and grouping (d, f, d_1), on one hand, and (q, q_1, q_2), on the other hand:

$$\begin{bmatrix} v_d \\ -v_f \\ 0 \end{bmatrix} = -\begin{bmatrix} R_a & & \\ & R_f & \\ & & R_{d1} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{d1} \end{bmatrix} - \begin{bmatrix} \dot{\theta}_r \psi_q \\ 0 \\ 0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{d1} \end{bmatrix}$$

$$\begin{bmatrix} v_q \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} R_a & & \\ & R_{q1} & \\ & & R_{q2} \end{bmatrix} \begin{bmatrix} i_q \\ i_{q1} \\ i_{q2} \end{bmatrix} + \begin{bmatrix} \dot{\theta}_r \psi_d \\ 0 \\ 0 \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \psi_q \\ \psi_{q1} \\ \psi_{q2} \end{bmatrix}$$

with the following flux-current relations:

$$\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_{d_1} \end{bmatrix} = \begin{bmatrix} L_{dd} & L_{df} & L_{dd1} \\ L_{df} & L_{ff} & L_{fd1} \\ L_{dd1} & L_{fd1} & L_{d1d1} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{d1} \end{bmatrix}$$

$$\begin{bmatrix} \psi_q \\ \psi_{q1} \\ \psi_{q2} \end{bmatrix} = \begin{bmatrix} L_{qq} & L_{qq_1} & L_{qq_2} \\ L_{qq1} & L_{q1q1} & L_{q1q2} \\ L_{qq2} & L_{q1q2} & L_{q2q2} \end{bmatrix} \begin{bmatrix} i_q \\ i_{q1} \\ i_{q2} \end{bmatrix}$$

Energy, power and torque

Balance of power at stator:

$$p_T + p_{Js} + \frac{dW_{ms}}{dt} = p_{r\rightarrow s}$$

p_T : three-phase instantaneous power leaving the stator
p_{Js} : Joule losses in stator windings
W_{ms} : magnetic energy stored in the stator windings
$p_{r\rightarrow s}$: power transfer from rotor to stator (mechanical ? electrical ?)

Three-phase instantaneous power leaving the stator :

$$\begin{aligned} p_T(t) &= v_a i_a + v_b i_b + v_c i_c = \boldsymbol{v}_T^T \boldsymbol{i}_T = \boldsymbol{v}_P^T \mathcal{P}\mathcal{P}^T \boldsymbol{i}_P = \boldsymbol{v}_P^T \boldsymbol{i}_P = v_d i_d + v_q i_q + v_o i_o \\ &= -\underbrace{(R_a i_d^2 + R_a i_q^2 + R_a i_o^2)}_{p_{Js}} - \underbrace{(i_d \frac{d\psi_d}{dt} + i_q \frac{d\psi_q}{dt} + i_o \frac{d\psi_o}{dt})}_{dW_{ms}/dt} + \dot{\theta}_r \, (\psi_d i_q - \psi_q i_d) \end{aligned}$$

$$\Rightarrow \quad p_{r\rightarrow s} = \dot{\theta}_r \, (\psi_d i_q - \psi_q i_d)$$

Balance of power at rotor:

$$P_m + p_f = p_{Jr} + \frac{dW_{mr}}{dt} + p_{r \to s} + \frac{dW_c}{dt}$$

P_m : mechanical power provided by the turbine
p_f : electrical power provided to the field winding (by the excitation system)
p_{Jr} : Joule losses in the rotor windings
W_{mr} : magnetic energy stored in the rotor windings
W_c : kinetic energy of all rotating masses.

Instantaneous power provided to field winding:

$$\begin{aligned} p_f &= v_f i_f = v_f i_f + v_{d1} i_{d1} + v_{q1} i_{q1} + v_{q2} i_{q2} \\ &= \underbrace{\left(R_f i_f^2 + R_{d1} i_{d1}^2 + R_{q1} i_{q1}^2 + R_{q2} i_{q2}^2\right)}_{p_{Jr}} + \underbrace{i_f \frac{d\psi_f}{dt} + i_{d1} \frac{d\psi_{d1}}{dt} + i_{q1} \frac{d\psi_{q1}}{dt} + i_{q2} \frac{d\psi_{q2}}{dt}}_{dW_{mr}/dt} \end{aligned}$$

$$P_m - \frac{dW_c}{dt} = \dot{\theta}_r (\psi_d i_q - \psi_q i_d)$$

Equation of rotor motion:

$$\mathcal{I}\frac{d^2\theta_r}{dt^2} = T_m - T_e$$

$\mathcal{I}$: moment of inertia of all the rotating masses
T_m : mechanical torque applied to the rotor by the turbine
T_e : electromagnetic torque applied to the rotor by the generator.

Multiplying the above equation by $\dot{\theta}_r$:

$$\mathcal{I}\,\dot{\theta}_r\,\ddot{\theta}_r = \dot{\theta}_r T_m - \dot{\theta}_r T_e$$

$$\frac{dW_c}{dt} = P_m - \dot{\theta}_r T_e$$

P_m : mechanical power provided by the turbine.

Hence, the (compact and elegant !) expression of the electromagnetic torque is:

$$T_e = \psi_d i_q - \psi_q i_d$$

Note. The power transfer $p_{r\to s}$ from rotor to stator is of mechanical nature only.

The various components of the torque T_e

$$T_e = L_{dd} i_d i_q + L_{df} i_f i_q + L_{dd1} i_{d1} i_q - L_{qq} i_q i_d - L_{qq1} i_{q1} i_d - L_{qq2} i_{q2} i_d$$

$(L_{dd} - L_{qq})\, i_d i_q$: *synchronous torque due to rotor saliency*

- exists in salient-pole machines only
- even without excitation $(i_f = 0)$, the rotor tends to align its direct axis with the axis of the rotating magnetic field created by the stator currents, offering to the latter a longer path in iron
- a significant fraction of the total torque in a salient-pole generator.

$L_{dd_1} i_{d1} i_q - L_{qq1} i_{q1} i_d - L_{qq2} i_{q2} i_d$: *damping torque*

- due to currents induced in the amortisseurs
- zero in steady-state operation.

$L_{df} i_f i_q$: only component involving the field current i_f

- the main part of the total torque in steady-state operation
- in steady state, it is the *synchronous torque due to excitation*
- during transients, the field winding also contributes to the damping torque.

The synchronous machine in steady state

- Balanced three-phase currents of angular frequency ω_N flow in the stator windings
- a direct current flows in the field winding subjected to a constant excitation voltage:

$$i_f = \frac{V_f}{R_f}$$

- the rotor rotates at the synchronous speed:

$$\theta_r = \theta_r^o + \omega_N t$$

- no current is induced in the other rotor circuits:

$$i_{d1} = i_{q1} = i_{q2} = 0$$

Operation with stator opened

$$
\begin{aligned}
i_a &= i_b = i_c = 0 \\
\Rightarrow \quad i_d &= i_q = i_o = 0 \\
\Rightarrow \quad \psi_d &= L_{df}\, i_f \quad \text{and} \quad \psi_q = 0
\end{aligned}
$$

Park equations:

$$
\begin{aligned}
v_d &= 0 \\
v_q &= \omega_N \psi_d = \omega_N L_{df}\, i_f
\end{aligned}
$$

Getting back to the stator voltages, e.g. in phase a :

$$
v_a(t) = \sqrt{\frac{2}{3}} \omega_N L_{df}\, i_f \sin(\theta_r^o + \omega_N t) = \sqrt{2} E_q \sin(\theta_r^o + \omega_N t)
$$

$$
E_q = \frac{\omega_N L_{df}\, i_f}{\sqrt{3}} = \text{e.m.f. proportional to excitation current}
$$
$$
= \text{RMS voltage at the terminal of the opened machine.}
$$

Operation under load

$$v_a(t) = \sqrt{2}V\cos(\omega_N t+\theta) \qquad v_b(t) = \sqrt{2}V\cos(\omega_N t+\theta-\frac{2\pi}{3}) \qquad v_c(t) = \sqrt{2}V\cos(\omega_N t+\theta+\frac{2\pi}{3})$$

$$i_a(t) = \sqrt{2}I\cos(\omega_N t+\psi) \qquad i_b(t) = \sqrt{2}I\cos(\omega_N t+\psi-\frac{2\pi}{3}) \qquad i_c(t) = \sqrt{2}I\cos(\omega_N t+\psi+\frac{2\pi}{3})$$

$$\begin{aligned} i_d &= \sqrt{\frac{2}{3}}\sqrt{2}I\,[\cos(\theta_r^o+\omega_N t)\cos(\omega_N t+\psi)+\cos(\theta_r^o+\omega_N t-\frac{2\pi}{3})\cos(\omega_N t+\psi-\frac{2\pi}{3}) \\ &\quad +\cos(\theta_r^o+\omega_N t+\frac{2\pi}{3})\cos(\omega_N t+\psi+\frac{2\pi}{3})] \\ &= \frac{I}{\sqrt{3}}[\cos(\theta_r^o+2\omega_N t+\psi)+\cos(\theta_r^o+2\omega_N t+\psi-\frac{4\pi}{3})+\cos(\theta_r^o+2\omega_N t+\psi+\frac{4\pi}{3}) \\ &\quad +3\cos(\theta_r^o-\psi)] = \sqrt{3}I\cos(\theta_r^o-\psi) \end{aligned}$$

Similarly:

$$\begin{aligned} i_q &= \sqrt{3}I\sin(\theta_r^o-\psi) & i_o &= 0 \\ v_d &= \sqrt{3}V\cos(\theta_r^o-\theta) & v_q &= \sqrt{3}V\sin(\theta_r^o-\theta) \qquad v_o = 0 \end{aligned}$$

In steady-state, i_d and i_q are constant. This was expected !

Magnetic flux in the d and q windings:

$$\begin{aligned} \psi_d &= L_{dd} i_d + L_{df} i_f \\ \psi_q &= L_{qq} i_q \end{aligned}$$

The electromagnetic torque:

$$T_e = \psi_d i_q - \psi_q i_d$$

is constant. This is important from mechanical viewpoint (no vibration !).

Park equations:

$$\begin{aligned} v_d &= -R_a i_d - \omega_N L_{qq} i_q = -R_a i_d - X_q i_q \\ v_q &= -R_a i_q + \omega_N L_{dd} i_d + \omega_N L_{df} i_f = -R_a i_q + X_d i_d + \sqrt{3} E_q \\ v_o &= 0 \end{aligned}$$

$X_d = \omega_N L_{dd}$: *direct-axis synchronous reactance*
$X_q = \omega_N L_{qq}$: *quadrature-axis synchronous reactance*

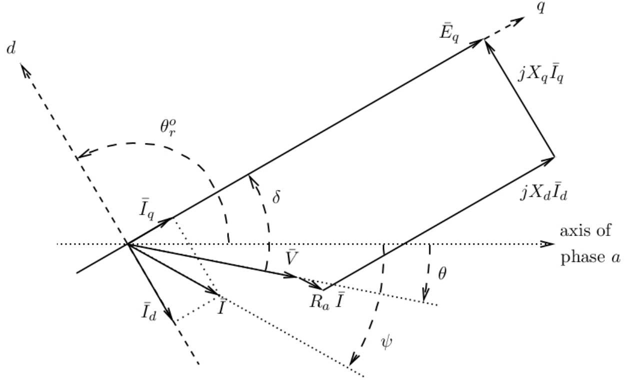
Phasor diagram

The Park equations become:

$$\begin{aligned} V\cos(\theta_r^o-\theta) &= -R_a I\cos(\theta_r^o-\psi) - X_q I\sin(\theta_r^o-\psi) \\ V\sin(\theta_r^o-\theta) &= -R_a I\sin(\theta_r^o-\psi) + X_d I\cos(\theta_r^o-\psi) + E_q \end{aligned}$$

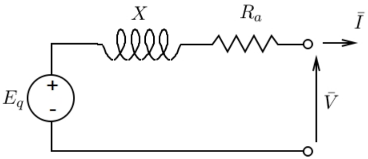
which are the projections on the d and q axes of the complex equation:

$$\bar{E}_q = \bar{V} + R_a\bar{I} + jX_d\bar{I}_d + jX_q\bar{I}_q$$



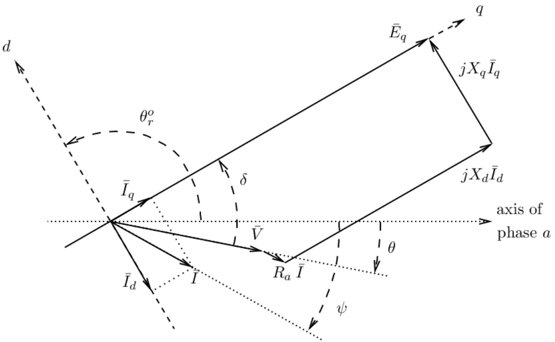
Particular case : round-rotor machine: $X_d = X_q = X$

$$\bar{E}_q = \bar{V} + R_a\bar{I} + jX(\bar{I}_d + \bar{I}_q) = \bar{V} + R_a\bar{I} + jX\bar{I}$$



Such an equivalent circuit cannot be derived for a salient-pole generator.

Powers



$$\bar{E}_q = E_q e^{j(\theta_r^o - \frac{\pi}{2})}$$

$$\bar{I}_d = I\cos(\theta_r^o - \psi)e^{j\theta_r^o} = \frac{i_d}{\sqrt{3}}e^{j\theta_r^o} \qquad \bar{I}_q = I\sin(\theta_r^o - \psi)e^{j(\theta_r^o - \frac{\pi}{2})} = -j\frac{i_q}{\sqrt{3}}e^{j\theta_r^o}$$

$$\bar{I} = \bar{I}_d + \bar{I}_q = (\frac{i_d}{\sqrt{3}} - j\frac{i_q}{\sqrt{3}})e^{j\theta_r^o}$$

$$\bar{V}_d = V\cos(\theta_r^o - \theta)e^{j\theta_r^o} = \frac{v_d}{\sqrt{3}}e^{j\theta_r^o} \qquad \bar{V}_q = V\sin(\theta_r^o - \theta)e^{j(\theta_r^o - \frac{\pi}{2})} = -j\frac{v_q}{\sqrt{3}}e^{j\theta_r^o}$$

$$\bar{V} = \bar{V}_d + \bar{V}_q = (\frac{v_d}{\sqrt{3}} - j\frac{v_q}{\sqrt{3}})e^{j\theta_r^o}$$

Three-phase complex power produced by the machine:

$$S = 3\bar{V}\bar{I}^{\star} = 3\,(\frac{v_d}{\sqrt{3}} - j\frac{v_q}{\sqrt{3}})(\frac{i_d}{\sqrt{3}} + j\frac{i_q}{\sqrt{3}}) = (v_d - j\,v_q)(i_d + j\,i_q)$$

$$\Rightarrow \quad P = v_d i_d + v_q i_q \qquad Q = v_d i_q - v_q i_d$$

P and Q as functions of V, E_q and the *internal angle* δ, assuming $R_a \simeq 0$?

$$\begin{aligned} v_d &= -X_q i_q \quad \Rightarrow \quad i_q = -\frac{v_d}{X_q} \\ v_q &= X_d i_d + \sqrt{3}E_q \quad \Rightarrow \quad i_d = \frac{v_q - \sqrt{3}E_q}{X_d} \\ v_d &= \sqrt{3}V\cos(\theta_r^o - \theta) \;=\; -\sqrt{3}V\sin\delta \\ v_q &= \sqrt{3}V\sin(\theta_r^o - \theta) \;=\; \sqrt{3}V\cos\delta \end{aligned}$$

$$P = 3\frac{E_q V}{X_d}\sin\delta + \frac{3V^2}{2}(\frac{1}{X_q} - \frac{1}{X_d})\sin 2\delta \qquad Q = 3\frac{E_q V}{X_d}\cos\delta - 3V^2(\frac{\sin^2\delta}{X_q} + \frac{\cos^2\delta}{X_d})$$

Case of a round-rotor machine: $P = 3\frac{E_q V}{X}\sin\delta \qquad Q = 3\frac{E_q V}{X}\cos\delta - 3\frac{V^2}{X}$

Nominal values, per unit system and orders of magnitudes

Stator

- nominal voltage U_N: voltage for which the machine has been designed (in particular its insulation).
  The real voltage may deviate from this value by a few %
- nominal current I_N: current for which machine has been designed (in particular the cross-section of its conductors).
  Maximum current that can be accepted without limit in time
- nominal apparent power: $S_N = \sqrt{3} U_N I_N$.

Conversion of parameters in per unit values:

- base power: $S_B = S_N$
- base voltage: $V_B = U_N / \sqrt{3}$
- base current: $I_B = S_N / 3V_B$
- base impedance: $Z_B = 3V_B^2 / S_B$.

Orders of magnitude

(more typical of machines with a nominal power above 100 MVA)
(pu values on the machine base)

	round-rotor machines	salient-pole machines
resistance R_a	0.005 pu	
direct-axis reactance X_d	1.5 - 2.5 pu	0.9 - 1.5 pu
quadrature-axis reactance X_q	1.5 - 2.5 pu	0.5 - 1.1 pu

Park (equivalent) windings

- base power: S_N
- base voltage: $\sqrt{3}V_B$
- base current: $\dfrac{S_N}{\sqrt{3}V_B} = \sqrt{3}I_B$ (single-phase formula !)

With this choice:

$$i_{dpu} = \frac{i_d}{\sqrt{3}I_B} = \frac{\sqrt{3}}{\sqrt{3}}\frac{I}{I_B}\cos(\theta_r^o - \psi) = I_{pu}\cos(\theta_r^o - \psi)$$

Similarly:

$$i_{qpu} = I_{pu}\sin(\theta_r^o - \psi) \qquad v_{dpu} = V_{pu}\cos(\theta_r^o - \theta) \qquad v_{qpu} = V_{pu}\sin(\theta_r^o - \theta)$$

$$\bar{I} = \bar{I}_d + \bar{I}_q = (i_d - j\, i_q)e^{j\theta_r^o} \qquad \bar{V} = \bar{V}_d + \bar{V}_q = (v_d - j\, v_q)e^{j\theta_r^o}$$

- All coefficients $\sqrt{3}$ have disappeared
- hence, the Park currents (resp. voltages) are exactly the projections on the machine d and q axes of the phasor $\bar{I}$ (resp. $\bar{V}$)