ELEC0029 - Electric power systems analysis and operation

Analysis of balanced faults

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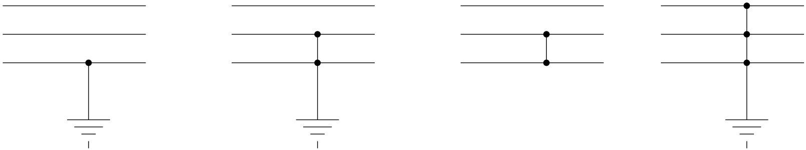
Types of faults



Single phase to ground faults :

- the most frequent.
  Example : Belgian 400-kV grid : 91 % of faults (2006-2014)

Three-phase faults :

- much less frequent. Example : Belgian 400-kV grid : only 2 % of faults
- but very often the most severe
- worst case that the network must be able to withstand.

This chapter focuses on three-phase faults with the same (in particular zero) resistance between each phase and the ground

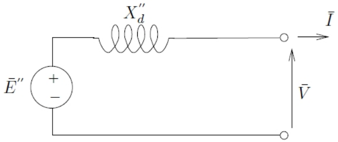
- the network remains electrically balanced → *balanced fault analysis*
- per-phase analysis can still be used.

Behaviour of the synchronous machine during a three-phase fault

Please refer to the separate lecture:

Behaviour of synchronous machine during a short-circuit
(a simple example of electromagnetic transients)

where the equivalent circuit of the machine is derived:



The stator resistance being small, it is usually neglected in short-circuit calculations

Computation of three-phase short-circuit currents

Motivation

- dimensioning the circuit breakers: breaking capability must be sufficient
- adjusting the settings of protections:
  - must not act in normal operation
  - must act in case of a fault.

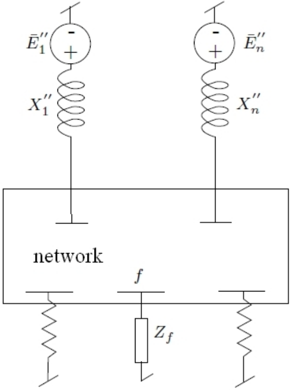
Assumptions

- network with N nodes
- n synchronous machines connected to nodes 1 to n (for simplicity)
- each synchronous machine represented by its equivalent circuit $(\bar{E}'', X'')$ (stator resistance neglected)
- loads represented by constant (shunt) admittances, obtained from the pre-fault operating conditions:

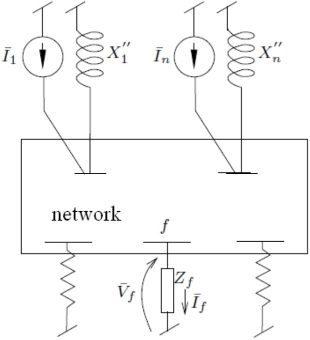
$$Y_c = \frac{P^{pre} - jQ^{pre}}{[V^{pre}]^2}$$

- node f subject to a short-circuit with impedance Z_f.

machines with Thévenin equivalents



machines with Norton equivalents



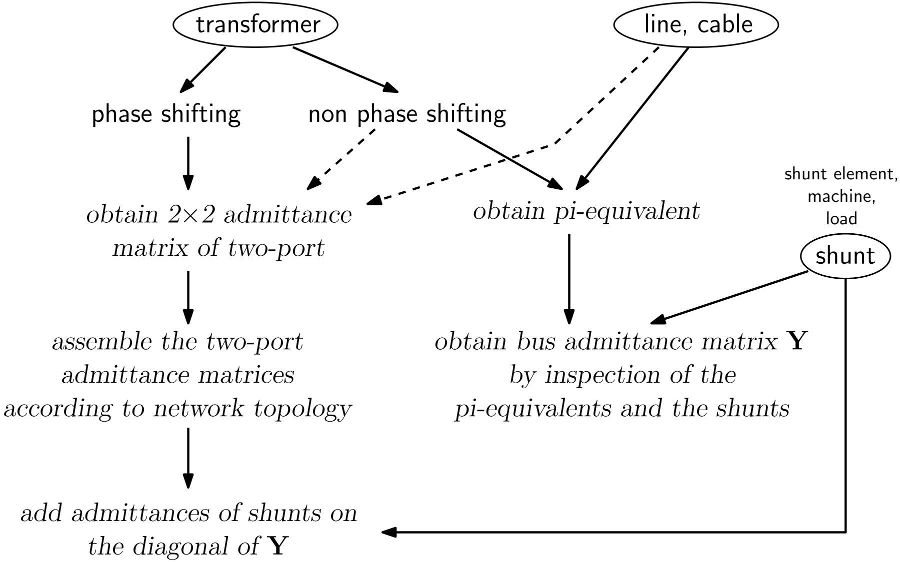
$$\bar{\boldsymbol{I}} = \boldsymbol{Y}\ \bar{\boldsymbol{V}}$$

$\bar{\boldsymbol{I}}$: vector of complex currents injected in the N buses,
each current counted positive if it enters the network

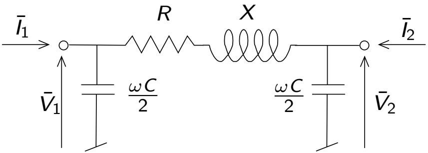
$\bar{\boldsymbol{V}}$: vector of complex voltages at the N buses

$\boldsymbol{Y}$: bus (or nodal) admittance matrix

Determination of the bus (or nodal) admittance matrix



Admittance matrix of the two-port representing a line or a cable

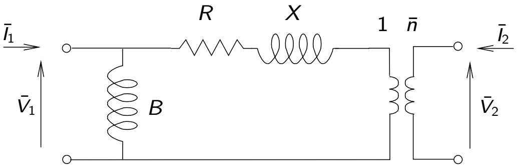


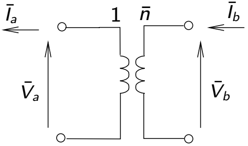
$$\begin{aligned}
\bar{I}_1 &= j\frac{\omega C}{2}\bar{V}_1 + \frac{1}{R+jX}(\bar{V}_1 - \bar{V}_2) = (j\frac{\omega C}{2} + \frac{1}{R+jX})\bar{V}_1 - \frac{1}{R+jX}\bar{V}_2 \\
\bar{I}_2 &= j\frac{\omega C}{2}\bar{V}_2 + \frac{1}{R+jX}(\bar{V}_2 - \bar{V}_1) = -\frac{1}{R+jX}\bar{V}_1 + (j\frac{\omega C}{2} + \frac{1}{R+jX})\bar{V}_1
\end{aligned}$$

and hence:

$$\mathbf{Y} = \begin{bmatrix} j\frac{\omega C}{2} + \frac{1}{R+jX} & -\frac{1}{R+jX} \\ -\frac{1}{R+jX} & j\frac{\omega C}{2} + \frac{1}{R+jX} \end{bmatrix}$$

Admittance matrix of the two-port representing a transformer
(including the case of a phase shifter)





$\bar{V}_b = \bar{n}\bar{V}_a \qquad \bar{I}_a = \bar{n}^\star \bar{I}_b$

$$\bar{I}_1 = jB\bar{V}_1 + \frac{1}{R+jX}(\bar{V}_1 - \frac{\bar{V}_2}{\bar{n}}) = (jB + \frac{1}{R+jX})\bar{V}_1 - \frac{1}{(R+jX)\bar{n}}\bar{V}_2$$

$$\bar{I}_2 = \frac{1}{\bar{n}^\star}(\frac{\bar{V}_2}{\bar{n}} - \bar{V}_1)\frac{1}{R+jX} = -\frac{1}{(R+jX)\bar{n}^\star}\bar{V}_1 + \frac{1}{(R+jX)n^2}\bar{V}_2$$

and hence:

$$\mathbf{Y} = \begin{bmatrix} jB + \dfrac{1}{R+jX} & -\dfrac{1}{(R+jX)\bar{n}} \\ -\dfrac{1}{(R+jX)\bar{n}^\star} & \dfrac{1}{(R+jX)n^2} \end{bmatrix}$$

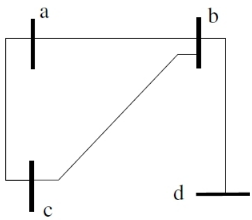
non symmetric if the transformer ratio is a complex number (phase shifter)

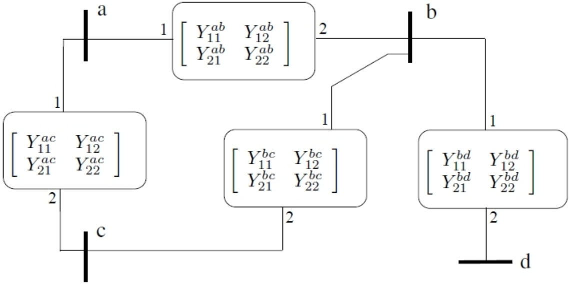
Assembling the two-port admittance matrices according to topology

For an N-bus system:

- initialize $\boldsymbol{Y}$ to a zero matrix of dimensions $N \times N$
- determine the (2×2) admittance matrices of the two-ports representing the lines, the cables and the transformers
- add the four terms of each of those matrices to the proper terms of $\boldsymbol{Y}$, taking into account the numbering of the terminal nodes
- add the admittances of machines, loads and shunt compensation elements to the corresponding diagonal terms of $\boldsymbol{Y}$.

Example on a four-bus network

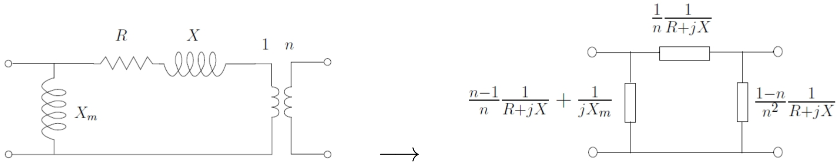




$$\mathbf{Y} = \begin{bmatrix} Y_{11}^{ab} + Y_{11}^{ac} & Y_{12}^{ab} & Y_{12}^{ac} & 0 \\ Y_{21}^{ab} & Y_{22}^{ab} + Y_{11}^{bc} + Y_{11}^{bd} & Y_{12}^{bc} & Y_{12}^{bd} \\ Y_{21}^{ac} & Y_{21}^{bc} & Y_{22}^{ac} + Y_{22}^{bc} & 0 \\ 0 & Y_{21}^{bd} & 0 & Y_{22}^{bd} \end{bmatrix}$$

Method by inspection

- Lines and cables are represented by their pi-equivalents
- the equivalent circuits of transformers *without phase shift* (real ratio n) are replaced by their corresponding pi-equivalents:



- those pi-equivalents are assembled according to the network topology
- together with the admittances of shunts

- the terms of $\boldsymbol{Y}$ are determined *by inspection* as follows:
  - the off-diagonal term $[\boldsymbol{Y}]_{ij}$ $(i,j=1,\ldots,N;\; i\neq j)$ is the sum of the admittances of all the elements connecting nodes i and j, sign changed
  - the diagonal term $[\boldsymbol{Y}]_{ii}$ $(i=1,\ldots,N)$ is the sum of the admittances of all the elements connected to node i.

Exercise. Use the method by inspection to check that the transformer pi-equivalent shown above has the (2×2) admittance matrix given in slide #8.

Computation of during-fault voltages by superposition

From the during-fault voltages, the current in any component can be computed.

The fault admittance $Y_f = \dfrac{1}{Z_f}$ is not included in $\boldsymbol{Y}$. Indeed:

- many possible fault locations have to be analyzed.
  Including Y_f in $\boldsymbol{Y}$ would require modifying $\boldsymbol{Y}$ for each fault location
- "solid" faults are considered (as worst cases): $Z_f = 0 \;\Rightarrow\; Y_f \rightarrow \infty$
  cannot be included in $\boldsymbol{Y}$!

The derivation which follows:

- uses the $\boldsymbol{Y}$ matrix relative to the pre-fault configuration of the network
- encompasses the case $Z_f = 0$.

Under the combined effect of $(\bar{I}_1, \bar{I}_2, \ldots, \bar{I}_n)$ and $\bar{I}_f$, the voltages $\bar{\boldsymbol{V}}$ are such that:

$$\boldsymbol{Y}\ \bar{\boldsymbol{V}} = \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_n \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ -\bar{I}_f \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

By superposition :

$$\bar{\boldsymbol{V}} = \bar{\boldsymbol{V}}^{pre} + \Delta\bar{\boldsymbol{V}} \tag{1}$$

with:

$$\boldsymbol{Y}\ \bar{\boldsymbol{V}}^{pre} = \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_n \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{Y}\ \Delta\bar{\boldsymbol{V}} = -\bar{I}_f \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = -\bar{I}_f\ \boldsymbol{e}_f$$

$\bar{\boldsymbol{V}}^{pre}$: vector of pre-fault bus voltages

$\Delta\bar{\boldsymbol{V}}$: correction accounting for the short-circuit

$\boldsymbol{e}_f$: unit vector (all zero components, except the f-th one equal to 1).

At this step, the value of $\bar{I}_f$ is still unknown...

Let us solve provisionally:

$$\boldsymbol{Y}\ \Delta\bar{\boldsymbol{V}}^1 = \boldsymbol{e}_f$$

We have:

$$\Delta\bar{\boldsymbol{V}} = -\bar{I}_f\ \Delta\bar{\boldsymbol{V}}^1$$

Introducing this result in (1):

$$\bar{\boldsymbol{V}} = \bar{\boldsymbol{V}}^{pre} - \bar{I}_f\ \Delta\bar{\boldsymbol{V}}^1$$

The f-th component of this relation is:

$$\bar{V}_f = \bar{V}_f^{pre} - \bar{I}_f\ \ \Delta\bar{V}_f^1$$

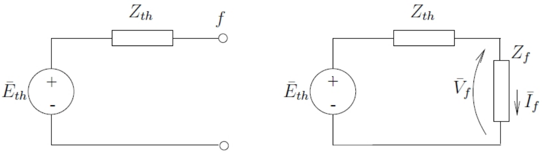
Combining with $\bar{V}_f = Z_f\ \bar{I}_f$ we obtain:

$$\bar{I}_f = \frac{\bar{V}_f^{pre}}{\Delta\bar{V}_f^1 + Z_f} \tag{2}$$

Substituting in (1), the during fault voltages are given by:

$$\bar{\boldsymbol{V}} = \bar{\boldsymbol{V}}^{pre} - \frac{\bar{V}_f^{pre}}{\Delta\bar{V}_f^1 + Z_f}\ \Delta\bar{\boldsymbol{V}}^1$$

Relation with Thévenin equivalent



Thévenin equivalent seen from node f:

- emf: $\bar{E}_{th} = \bar{V}_f^{pre}$
- impedance: Z_{th} = voltage at node f when a unit current is injected in this node, after the other current injectors have been replaced by open-circuits. Thus:

$$Z_{th} = \left[\mathbf{Y}^{-1}\mathbf{e}_f\right]_f = \left[\Delta\bar{\mathbf{V}}^1\right]_f = \Delta\bar{V}_f^1$$

Fault current: $\bar{I}_f = \dfrac{\bar{E}_{th}}{Z_{th} + Z_f} = \dfrac{\bar{V}_f^{pre}}{\Delta\bar{V}_f^1 + Z_f}$ which is nothing but Eq. (2)

Another way to obtain Z_{th} : $Z_{th} = \left[\mathbf{Y}^{-1}\mathbf{e}_f\right]_f = \left[\mathbf{Y}^{-1}\right]_{ff} = [\mathbf{Z}]_{ff}$

$\mathbf{Z}$ is called the *nodal impedance matrix*