ELEC0029 - Electric power systems analysis

Three-phase analysis of unbalanced systems

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The "traditional" approach

(see Homework # 3)

- Assemble the positive-sequence models of the various components (lines, cables, transformers, loas, generators) according to the network topology
- do the same with the negative-sequence models
- if needed[1], do the same with the zero-sequence models
- connect the positive-, negative- and zero-sequence models at the location of the fault, taking into the nature of this fault (see slides # 43-45 of the lecture on Symmetrical components)
- solve the resulting circuit for voltages and currents
- using Fortescue transformation, obtain the corresponding phase voltages and currents.

[1] not for a line-to-line fault

The approach followed in this lecture

- For lines, cables, transformers and generators, we consider how to pass from the symmetric-component model $(+, -, o)$ to the three-phase model (a, b, c)
- for the other components, we derive the (a, b, c) model directly
- all these individual models are assembled according to the network topology to build the three-phase bus admittance matrix $\boldsymbol{Y}$ and the vector of three-phase injected currents $\bar{\boldsymbol{I}}$ of:

$$\bar{\boldsymbol{I}} = \boldsymbol{Y}\bar{\boldsymbol{V}} \tag{1}$$

- the fault is taken into account by adding fault admittances to the proper terms of $\boldsymbol{Y}$
- the linear system (1) is solved to obtain the vector $\bar{\boldsymbol{V}}$ of phase voltages
- from which the currents in (a, b, c) branches are obtained.

For each component, a MATLAB script is available and its use is described.

Preliminary remarks

- All voltages refer to their "local grounds"
- as in the lecture on Symmetrical components, the neutrals are eliminated
- thus, for an N-bus system, $\boldsymbol{Y}$ is a $3N \times 3N$ matrix
- some models require values from an initial power flow computation of the balanced system
- since we are dealing with all three phases[2], the per unit system uses the single-phase base power (see course ELEC0014)

three-phase (S_{B3}, V_B)	single-phase $(S_{B1} = S_{B3}/3, V_B)$
$S_{B3} = 3V_B I_{B3} = \sqrt{3} U_B I_{B3}$	$S_{B1} = V_B I_{B1}$
$I_{B3} = \dfrac{S_{B3}}{3V_B} = \dfrac{S_{B3}}{\sqrt{3} U_B}$	$I_{B1} = \dfrac{S_{B1}}{V_B} = I_{B3}$
$Z_{B3} = \dfrac{V_B}{I_{B3}} = \dfrac{3V_B^2}{S_{B3}} = \dfrac{U_B^2}{S_{B3}}$	$Z_{B1} = \dfrac{V_B}{I_{B1}} = \dfrac{V_B^2}{S_{B1}} = Z_{B3}$

- the currents and impedances have the same values in per unit in both bases.

[2] and no longer with a per-phase analysis

Numerical example

Consider the generator of Homework # 3 :
- nominal apparent power : 5 MVA
- positive-sequence reactance X_+ : 0.15 pu on the generator three-phase base.

What is the value of X_+ on a single-phase base of 1 MVA ?

X_+ has the same value in per unit on both :
- the three-phase base power of 5 MVA
- the single-phase base power of 5/3 MVA.

To change to the single-phase base power of 1 MVA, apply the general formula of change of base given in course ELEC0014 :

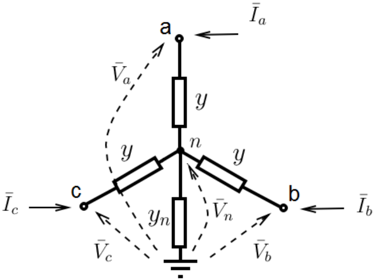
$$Z_{pu2} = Z_{pu1} \frac{S_{B2}}{S_{B1}} \left(\frac{V_{B1}}{V_{B2}} \right)^2$$

and the requested value is :

$$0.15 \frac{1}{5/3} = 0.15 \frac{3}{5} = 0.09 \quad \text{pu}$$

Load

Star-connected load



Input data:

- complex power consumed in one phase: $P + jQ$ (in pu)
- magnitude of the phase-to-neutral voltage under which this power is consumed: V (in pu)
- impedance between neutral and ground: z_n (in pu)

$$y = \frac{P - jQ}{V^2} \qquad y_n = \frac{1}{z_n}$$

$$\begin{aligned} \bar{I}_a &= y(\bar{V}_a - \bar{V}_n) && (2)\\ \bar{I}_b &= y(\bar{V}_b - \bar{V}_n) && (3)\\ \bar{I}_c &= y(\bar{V}_c - \bar{V}_n) && (4)\\ \bar{I}_a + \bar{I}_b + \bar{I}_c &= y_n \bar{V}_n && (5) \end{aligned}$$

Adding (2, 3 and 4) and introducing the result in (5) yields:

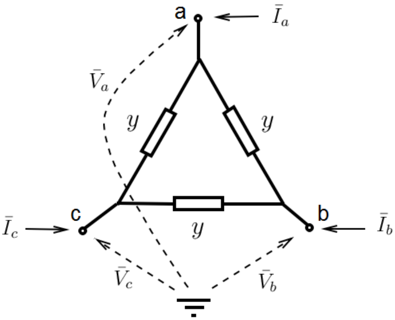
$$\bar{V}_n = \frac{y\bar{V}_a + y\bar{V}_b + y\bar{V}_c}{y_{tot}} \quad \text{with} \quad y_{tot} = 3y + y_n \qquad (6)$$

Introducing (6) into (2,3,4) and arranging the results in matrix form:

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \underbrace{\begin{bmatrix} y - \frac{y^2}{y_{tot}} & -\frac{y^2}{y_{tot}} & -\frac{y^2}{y_{tot}} \\ -\frac{y^2}{y_{tot}} & y - \frac{y^2}{y_{tot}} & -\frac{y^2}{y_{tot}} \\ -\frac{y^2}{y_{tot}} & -\frac{y2}{y_{tot}} & y - \frac{y^2}{y_{tot}} \end{bmatrix}}_{\text{contribution to matrix } \boldsymbol{Y}} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

Question. Determine the admittance matrix when $z_n = 0$ ("solid" grounding)?

Delta-connected load



Input data:

- complex power consumed in each branch: $P + jQ$ (in pu)
- magnitude of the phase-to-phase voltage under which those powers are consumed: U (in pu)

$$y = \frac{P - jQ}{U^2}$$

$$\begin{aligned}\bar{I}_a &= y(\bar{V}_a - \bar{V}_b) + y(\bar{V}_a - \bar{V}_c) \\ \bar{I}_b &= y(\bar{V}_b - \bar{V}_a) + y(\bar{V}_b - \bar{V}_c) \\ \bar{I}_c &= y(\bar{V}_c - \bar{V}_a) + y(\bar{V}_c - \bar{V}_b)\end{aligned}$$

which can be rewritten in matrix form as:

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \underbrace{\begin{bmatrix} 2y & -y & -y \\ -y & 2y & -y \\ -y & -y & 2y \end{bmatrix}}_{\text{contribution to matrix } \boldsymbol{Y}} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

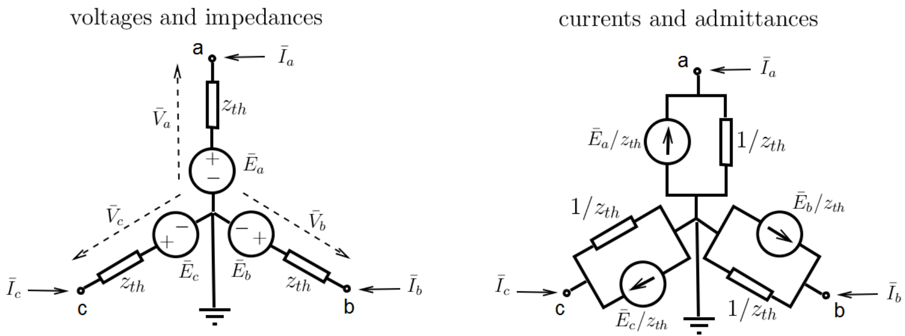
Remarks

- this admittance matrix is singular because the voltage reference node is not located in the circuit ("*indefinite admittance matrix*")
- this is not a problem since it will be combined with other admittance matrices to form $\boldsymbol{Y}$.

Matlab script Yload.m

```
function [Yld] = Yload(triangle,rn,xn,p,q,v)

% triangle= 1 if the load is assembled in triangle, =0 if assembled in star
% rn : neutral resistance (in pu), not used if triangle=1
% xn : neutral  reactance (in pu), not used if triangle=1
%      set rn and xn to a very large value if the neutral is not grounded
% p : active power (in pu) consumed:
%           in one branch of the triangle (if triangle=1)
%           in one branch of the star (if triangle=0)
% q : reactive power (in pu) consumed:
%           in one branch of the triangle (if triangle=1)
%           in one branch of the star (if triangle=0)
% v : magnitude of voltage (in pu) under which the above powers are consumed
%     phase-to-phase voltage if triangle=1, phase-to-neutral if triangle=0

% Yld : 3x3 admittance matrix of three-phase load
```

Balanced Norton equivalent



Input data:

- complex power produced by one of the three phases: $P + jQ$ (in pu)
- Thévenin impedance: z_{th} (in pu)
- complex (phase-to-neutral) voltage of phase a: $\bar{V}_a$ (in pu)

$$\bar{E}_a = \bar{V}_a + z_{th}\frac{P - jQ}{\bar{V}_a^{\star}} \qquad \bar{E}_b = a^2\bar{E}_a \qquad \bar{E}_c = a\bar{E}_a$$

$$
\begin{aligned}
\bar{I}_a &= (\bar{V}_a - \bar{E}_a)/z_{th} \\
\bar{I}_b &= (\bar{V}_b - \bar{E}_b)/z_{th} \\
\bar{I}_c &= (\bar{V}_c - \bar{E}_c)/z_{th}
\end{aligned}
$$

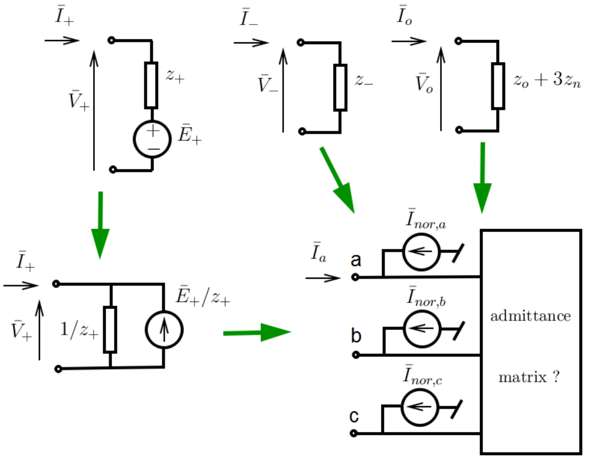
which can be rewritten in matrix form as:

$$
\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \underbrace{\begin{bmatrix} 1/z_{th} & 0 & 0 \\ 0 & 1/z_{th} & 0 \\ 0 & 0 & 1/z_{th} \end{bmatrix}}_{\text{contribution to matrix } \boldsymbol{Y}} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} - \underbrace{\begin{bmatrix} \bar{E}_a/z_{th} \\ \bar{E}_b/z_{th} \\ \bar{E}_c/z_{th} \end{bmatrix}}_{\text{contrib. to vector } \bar{\boldsymbol{I}}}
$$

Matlab script Norton_equ.m

```
function [Ynor,Inor] = Norton_equ(rth,xth,p,q,v,theta)

% rth : Thévenin equivalent resistance (in pu)
% xth : Thévenin equivalent  reactance (in pu)
% p :   active power produced by one phase (in pu)
% q : reactive power produced by one phase (in pu)
% v : magnitude of (phase-to-ground) voltage of phase a (pu)
% theta : phase angle of (phase-to-ground) voltage of phase a (rad)

% Ynor : 3x3 admittance matrix of Norton equivalent
% Inor : 3x1 vector of Norton currents
```

Generator



Input data:

- positive, negative and zero-sequence impedances: z_+, z_- and z_o (in pu)
- impedance between neutral and ground: z_n (in pu)
- complex power produced by one of the three phases: $P + jQ$ (in pu)
- complex (phase-to-neutral) voltage of phase a: $\bar{V}_a$ (in pu)

To identify $\bar{I}_{nor,a}$, $\bar{I}_{nor,b}$ and $\bar{I}_{nor,c}$ we assume initial balanced operating conditions:

$$\bar{V}_- = \bar{V}_o = 0 \qquad \bar{V}_+ = \bar{V}_a \qquad \bar{I}_- = \bar{I}_o = 0 \qquad \bar{I}_+ = \bar{I}_a = -\frac{P - jQ}{\bar{V}_a^\star}$$

Hence:

$$\bar{E}_+ = \bar{V}_+ - z_+ \bar{I}_+ = \bar{V}_a + z_+ \frac{P - jQ}{\bar{V}_a^\star}$$

Contribution to vector $\bar{\boldsymbol{I}}$:

$$\begin{bmatrix} \bar{I}_{nor,a} \\ \bar{I}_{nor,b} \\ \bar{I}_{nor,c} \end{bmatrix} = \begin{bmatrix} \bar{E}_+/z_+ \\ a^2\, \bar{E}_+/z_+ \\ a\, \bar{E}_+/z_+ \end{bmatrix}$$

Contribution to admittance matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y}_T = \boldsymbol{T}\boldsymbol{Y}_F\boldsymbol{T}^{-1}$$

with:

$$\boldsymbol{Y}_F = \begin{bmatrix} 1/z_+ & 0 & 0 \\ 0 & 1/z_- & 0 \\ 0 & 0 & 1/(z_o + 3z_n) \end{bmatrix} \quad \text{and} \quad \boldsymbol{T} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix}$$

Matlab script Norton_gen.m

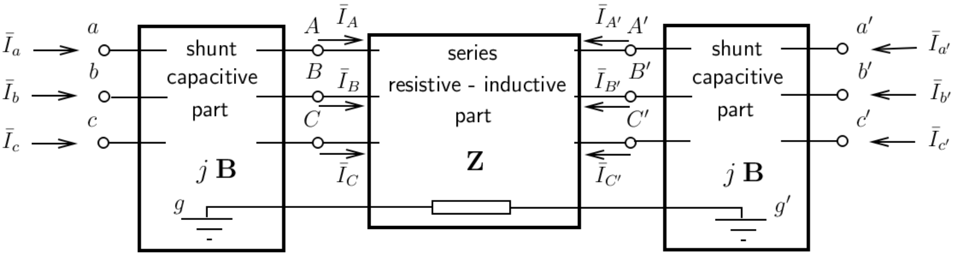
```
function [Ygen,Igen] = Norton_gen(rp,xp,rm,xm,r0,x0,rn,xn,p,q,v,theta)

% rp : direct-sequence resistance (pu)
% xp : direct-sequence reactance  (pu)
% rm : negative-sequence resistance (pu)
% xm : negative-sequence reactance (pu)
% r0 : zero-sequence resistance (pu)
% x0 : zero-sequence reactance (pu)
% rn : neutral resistance (pu)
% xn : neutral reactance (pu)
%      set rn and xn to a very large value if neutral is not grounded
% p : active power produced by one phase of the generator (pu)
% q : reactive power produced by one phase of the generator (pu)
% v : voltage magnitude (pu), initial value in phase a
% theta : voltage phase angle (rad), initial value in phase a

% Ygen : 3x3 admittance matrix of generator
% Igen : 3x1 vector of Norton equivalent currents (INTO the network)
```

Line or cable

Full three-phase symmetry is assumed.

Input data:

- positive-sequence series impedance: $z_+ = r_+ + jx_+$ (in pu)
- zero-sequence series impedance: $z_o = r_o + jx_o$ (in pu)
- positive-sequence half shunt susceptance: b_+ (in pu)
- zero-sequence half shunt susceptance: b_o (in pu)



It has been shown (see theory): $z_+ = z_s - z_m \qquad z_o = z_s + 2z_m$

from which one obtains: $z_s = \dfrac{2z_+ + z_o}{3} \qquad z_m = \dfrac{z_o - z_+}{3}$

and the impedance matrix: $\boldsymbol{Z} = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}$

The voltage-current relations of the series part are:

$$\begin{bmatrix} \overline{V}_A - \overline{V}_g \\ \overline{V}_B - \overline{V}_g \\ \overline{V}_C - \overline{V}_g \end{bmatrix} = \boldsymbol{Z} \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix} + \begin{bmatrix} \overline{V}_{A'} - \overline{V}_{g'} \\ \overline{V}_{B'} - \overline{V}_{g'} \\ \overline{V}_{C'} - \overline{V}_{g'} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix} = \boldsymbol{Z}^{-1} \begin{bmatrix} \overline{V}_A - \overline{V}_g \\ \overline{V}_B - \overline{V}_g \\ \overline{V}_C - \overline{V}_g \end{bmatrix} - \boldsymbol{Z}^{-1} \begin{bmatrix} \overline{V}_{A'} - \overline{V}_{g'} \\ \overline{V}_{B'} - \overline{V}_{g'} \\ \overline{V}_{C'} - \overline{V}_{g'} \end{bmatrix} \qquad (7)$$

and
$$\begin{bmatrix} \overline{I}_{A'} \\ \overline{I}_{B'} \\ \overline{I}_{C'} \end{bmatrix} = - \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix} = -\boldsymbol{Z}^{-1} \begin{bmatrix} \overline{V}_A - \overline{V}_g \\ \overline{V}_B - \overline{V}_g \\ \overline{V}_C - \overline{V}_g \end{bmatrix} + \boldsymbol{Z}^{-1} \begin{bmatrix} \overline{V}_{A'} - \overline{V}_{g'} \\ \overline{V}_{B'} - \overline{V}_{g'} \\ \overline{V}_{C'} - \overline{V}_{g'} \end{bmatrix} \qquad (8)$$

It has been shown (see theory): $b_+ = b_s - b_m \qquad b_o = b_s + 2b_m$

from which one obtains: $b_s = \dfrac{2b_+ + b_o}{3} \qquad b_m = \dfrac{b_o - b_+}{3}$

and the admittance matrix: $j\boldsymbol{B} = j \begin{bmatrix} b_s & b_m & b_m \\ b_m & b_s & b_m \\ b_m & b_m & b_s \end{bmatrix}$

For the shunt part on the left, the voltage-current relations are:

$$\begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix} = j\,\boldsymbol{B} \begin{bmatrix} \overline{V}_a - \overline{V}_g \\ \overline{V}_b - \overline{V}_g \\ \overline{V}_c - \overline{V}_g \end{bmatrix} + \begin{bmatrix} \overline{I}_A \\ \overline{I}_B \\ \overline{I}_C \end{bmatrix} \tag{9}$$

Similarly, for the shunt part on the right:

$$\begin{bmatrix} \overline{I}_{a'} \\ \overline{I}_{b'} \\ \overline{I}_{c'} \end{bmatrix} = j\,\boldsymbol{B} \begin{bmatrix} \overline{V}_{a'} - \overline{V}_g \\ \overline{V}_{b'} - \overline{V}_g \\ \overline{V}_{c'} - \overline{V}_g \end{bmatrix} + \begin{bmatrix} \overline{I}_{A'} \\ \overline{I}_{B'} \\ \overline{I}_{C'} \end{bmatrix} \tag{10}$$

Taking into account that:

$$\overline{V}_a = \overline{V}_A \quad \overline{V}_b = \overline{V}_B \quad \overline{V}_c = \overline{V}_C \quad \overline{V}_{a'} = \overline{V}_{A'} \quad \overline{V}_{b'} = \overline{V}_{B'} \quad \overline{V}_{c'} = \overline{V}_{C'}$$

Eqs. (7, 8, 9, 10) can be combined into:

$$\begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \\ \hline \overline{I}_{a'} \\ \overline{I}_{b'} \\ \overline{I}_{c'} \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} j\boldsymbol{B} + \boldsymbol{Z}^{-1} & -\boldsymbol{Z}^{-1} \\ \hline -\boldsymbol{Z}^{-1} & j\boldsymbol{B} + \boldsymbol{Z}^{-1} \end{array}\right]}_{6\times 6 \text{ admittance matrix } \boldsymbol{Y}} \begin{bmatrix} \overline{V}_a - \overline{V}_g \\ \overline{V}_b - \overline{V}_g \\ \overline{V}_c - \overline{V}_g \\ \hline \overline{V}_{a'} - \overline{V}_{g'} \\ \overline{V}_{b'} - \overline{V}_{g'} \\ \overline{V}_{c'} - \overline{V}_{g'} \end{bmatrix}$$

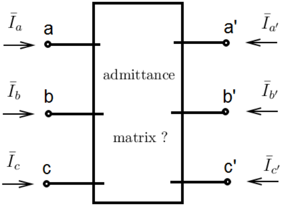
Matlab script Yline.m

```
function [Yln] = Yline (rp,xp,bp,r0,x0,b0)

% rp : direct-sequence series resistance (pu)
% xp : direct-sequence series reactance (pu)
% bp : direct-sequence half shunt susceptance (pu)
% r0 : zero-sequence series resistance (pu)
% x0 : zero-sequence series reactance (pu)
% b0 : zero-sequence half shunt susceptance (pu)

% Yln : 6x6 admittance matrix of line
```

Transformer

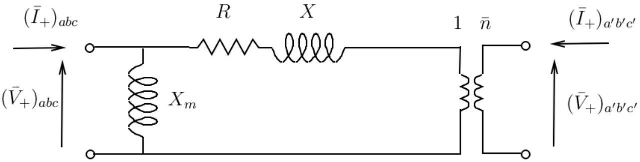
Input data:

- positive-sequence series impedance: $R + jX$ (in pu)
- positive-sequence magnetizing reactance: X_m (in pu)
- zero-sequence series impedance: $R_o + jX_o$ (in pu)
- zero-sequence magnetizing reactance: X_{mo} (in pu)
- impedance neutral - ground on primary and secondary sides: z_{n1}, z_{n2} (in pu) [3]
- complex transformer ratio $\bar{n}$



[3] not used in some transformers

Admittance matrix of the positive-sequence equivalent two-port



$$
\begin{aligned}
(\bar{I}_+)_{abc} &= \frac{1}{jX_m}(\bar{V}_+)_{abc} + \frac{1}{R+jX}\left[(\bar{V}_+)_{abc} - \frac{1}{\bar{n}}(\bar{V}_+)_{a'b'c'}\right] \\
(\bar{I}_+)_{a'b'c'} &= \frac{1}{\bar{n}^\star}\frac{1}{R+jX}\left[\frac{1}{\bar{n}}(\bar{V}_+)_{a'b'c'} - (\bar{V}_+)_{abc}\right]
\end{aligned}
$$

or in matrix form:

$$
\begin{bmatrix} (\bar{I}_+)_{abc} \\ (\bar{I}_+)_{a'b'c'} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{jX_m} + \frac{1}{R+jX} & -\frac{1}{\bar{n}}\frac{1}{R+jX} \\ -\frac{1}{\bar{n}^\star}\frac{1}{R+jX} & \frac{1}{|\bar{n}|^2}\frac{1}{R+jX} \end{bmatrix}}_{\text{posit.-sequ. admittance matrix}} \begin{bmatrix} (\bar{V}_+)_{abc} \\ (\bar{V}_+)_{a'b'c'} \end{bmatrix}
$$

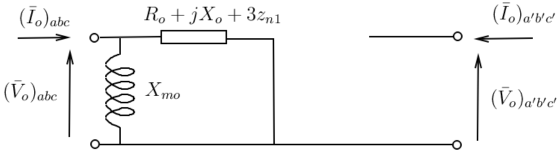
Admittance matrix of the negative-sequence equivalent two-port

Replacing $\bar{n}$ by $\bar{n}^\star$ in the positive-sequence admittance matrix:

$$\begin{bmatrix} (\bar{I}_-)_{abc} \\ (\bar{I}_-)_{a'b'c'} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{jX_m} + \frac{1}{R+jX} & -\frac{1}{\bar{n}^\star}\frac{1}{R+jX} \\ -\frac{1}{\bar{n}}\frac{1}{R+jX} & \frac{1}{|\bar{n}|^2}\frac{1}{R+jX} \end{bmatrix}}_{\text{negative-sequ. admittance matrix}} \begin{bmatrix} (\bar{V}_-)_{abc} \\ (\bar{V}_-)_{a'b'c'} \end{bmatrix}$$

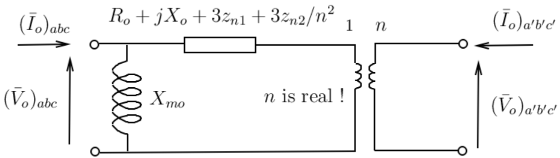
Admittance matrix of the zero-sequence equivalent two-port

1. Transformer of type Ynd*



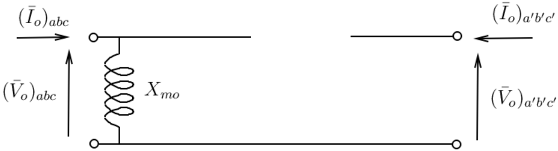
$$\begin{bmatrix} (\bar{I}_o)_{abc} \\ (\bar{I}_o)_{a'b'c'} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{jX_{mo}} + \frac{1}{R_o+jX_o+3z_{n1}} & 0 \\ 0 & 0 \end{bmatrix}}_{\text{zero-sequ. admit. matrix}} \begin{bmatrix} (\bar{V}_o)_{abc} \\ (\bar{V}_o)_{a'b'c'} \end{bmatrix}$$

2. Transformer of type Ynyn0



$$\underbrace{\begin{bmatrix} (\bar{I}_o)_{abc} \\ (\bar{I}_o)_{a'b'c'} \end{bmatrix} = \begin{bmatrix} \frac{1}{jX_{mo}} + \frac{1}{R_o+jX_o+3z_{n1}+3z_{n2}/n^2} & -\frac{1}{n}\frac{1}{R_o+jX_o+3z_{n1}+3z_{n2}/n^2} \\ -\frac{1}{n}\frac{1}{R_o+jX_o+3z_{n1}+3z_{n2}/n^2} & \frac{1}{n^2}\frac{1}{R_o+jX_o+3z_{n1}+3z_{n2}/n^2} \end{bmatrix}}_{\text{zero-sequ. admittance matrix}} \begin{bmatrix} (\bar{V}_o)_{abc} \\ (\bar{V}_o)_{a'b'c'} \end{bmatrix}$$

3. Transformer of any other type



$$\begin{bmatrix} (\bar{I}_o)_{abc} \\ (\bar{I}_o)_{a'b'c'} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{jX_{mo}} & 0 \\ 0 & 0 \end{bmatrix}}_{\text{zero-sequ. admit. matrix}} \begin{bmatrix} (\bar{V}_o)_{abc} \\ (\bar{V}_o)_{a'b'c'} \end{bmatrix}$$

Assembling the admittance matrices of all three sequences

$$\begin{bmatrix} (\bar{I}_+)_{abc} \\ (\bar{I}_-)_{abc} \\ (\bar{I}_o)_{abc} \\ (\bar{I}_+)_{a'b'c'} \\ (\bar{I}_-)_{a'b'c'} \\ (\bar{I}_o)_{a'b'c'} \end{bmatrix} = \boldsymbol{Y}_F \begin{bmatrix} (\bar{V}_+)_{abc} \\ (\bar{V}_-)_{abc} \\ (\bar{V}_o)_{abc} \\ (\bar{V}_+)_{a'b'c'} \\ (\bar{V}_-)_{a'b'c'} \\ (\bar{V}_o)_{a'b'c'} \end{bmatrix}$$

with

$$\boldsymbol{Y}_F = \begin{bmatrix} \frac{1}{jX_m} + \frac{1}{R+jX} & 0 & 0 & -\frac{1}{\bar{n}}\frac{1}{R+jX} & 0 & 0 \\ 0 & \frac{1}{jX_m} + \frac{1}{R+jX} & 0 & 0 & -\frac{1}{\bar{n}^\star}\frac{1}{R+jX} & 0 \\ 0 & 0 & y_{11} & 0 & 0 & y_{12} \\ -\frac{1}{\bar{n}^\star}\frac{1}{R+jX} & 0 & 0 & \frac{1}{|\bar{n}|^2}\frac{1}{R+jX} & 0 & 0 \\ 0 & -\frac{1}{\bar{n}}\frac{1}{R+jX} & 0 & 0 & \frac{1}{|\bar{n}|^2}\frac{1}{R+jX} & 0 \\ 0 & 0 & y_{21} & 0 & 0 & y_{22} \end{bmatrix}$$

where $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$ is the zero-sequence admittance matrix previously derived.

Getting back to the a, b, c, a', b', c' phases

Using the inverse Fortescue transformation:

$$\begin{bmatrix} \boldsymbol{T}^{-1} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} \\ \boldsymbol{T}^{-1} \begin{bmatrix} \bar{I}_{a'} \\ \bar{I}_{b'} \\ \bar{I}_{c'} \end{bmatrix} \end{bmatrix} = \boldsymbol{Y}_F \begin{bmatrix} \boldsymbol{T}^{-1} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} \\ \boldsymbol{T}^{-1} \begin{bmatrix} \bar{V}_{a'} \\ \bar{V}_{b'} \\ \bar{V}_{c'} \end{bmatrix} \end{bmatrix}$$

or

$$\left[\begin{array}{c} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \\ \hline \bar{I}_{a'} \\ \bar{I}_{b'} \\ \bar{I}_{c'} \end{array}\right] = \underbrace{\left[\begin{array}{c|c} \boldsymbol{T} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \boldsymbol{T} \end{array}\right] \boldsymbol{Y}_F \left[\begin{array}{c|c} \boldsymbol{T}^{-1} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \boldsymbol{T}^{-1} \end{array}\right]}_{\text{the sought admittance matrix. Yeah!}} \left[\begin{array}{c} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \\ \hline \bar{V}_{a'} \\ \bar{V}_{b'} \\ \bar{V}_{c'} \end{array}\right]$$

Matlab script Ytrfo.m

```
function [Ytf] = Ytrfo(rp,xp,bp,n,phi,b0,type,r0,x0,rn1,xn1,rn2,xn2)

% rp : positive-sequence series resistance (pu)
% xp : positive-sequence series reactance (pu)
% bp : positive-sequence magnetizing susceptance (pu)
% n  : magnitude of transformer ratio (pu/pu)
% phi : phase angle of transformer ratio (rad)
% b0 : zero-sequence magnetizing susceptance (pu)
% type = 1 indicates that the transformer is of the type Ynd
%      = 2 indicates that the transformer is of the type Ynyn
%      = 3 indicates that the transformer is of another type
%
% r0 : zero-sequence series resistance (pu); not used if type=3
% x0 : zero-sequence series  reactance (pu); not used if type=3
% rn1 : neutral resistance on the primary side (pu); not used if type=3
% xn1 : neutral  reactance on the primary side (pu); not used if type=3
% rn2 : neutral resistance on the secondary side (pu); not used if type=1 or 3
% xn2 : neutral  reactance on the secondary side (pu); not used if type=1 or 3
%
% set rn1 and xn1 (or rn2 and xn2) to very large values if neutral is not grounded

% Ytf : 6 x 6 admittance matrix of the transformer
```