ELEC0014 - *Introduction to electric power and energy systems*

The power transformer

Thierry Van Cutsem
t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

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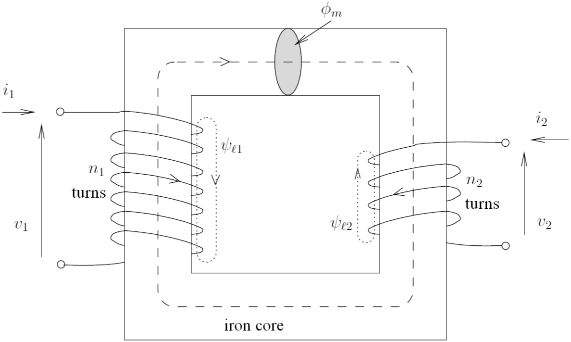
Power transformers are used:

- to transmit electrical energy under high voltages
  - step-up transformers at the terminal of generators
  - step-down transformers to distribute energy at the end-users
- to control the voltages at some busbars:
  - in sub-transmission networks
  - in distribution networks
- to control the power flows in some parts of a meshed network.

The single-phase transformer

Principle



step-up transformer: secondary voltage > primary voltage
step-down transformer: secondary voltage < primary voltage.

alternating voltage v_1 at terminals of coil 1 $\longrightarrow$ alternating current i_1 in coil 1 $\longrightarrow$ alternating magnetic field $\longrightarrow$ voltage induced in coil 2 $\longrightarrow$ current i_2 in coil 2 $\longrightarrow$ magnetic field superposed to the one created by i_1.

Fluxes in coils

$$n_1 i_1 + n_2 i_2 = \mathcal{R} \phi_m$$

ϕ_m : magnetic flux in a cross section of the iron core
$\mathcal{R}$: *reluctance* of the magnetic circuit

$+$ sign: due to the way coils are wound and the direction of currents

$$\psi_1 = \psi_{\ell 1} + n_1 \phi_m$$

ψ_1 : flux linkage in coil 1
$\psi_{\ell 1}$: *leakage flux* in coil 1 (lines of magnetic field crossing coil 1 but not passing through the iron core)

$$\psi_2 = \psi_{\ell 2} + n_2 \phi_m$$

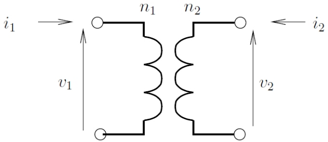
ψ_2 : flux linkage in coil 2
$\psi_{\ell 2}$: *leakage flux* in coil 2 (lines of magnetic field crossing coil 2 but not passing through the iron core)

Ideal transformer

- The coils have no resistance
- the coils have no leakage flux
- the permeability of the core material is infinite.

$$\mathcal{R} \simeq 0 \quad \Rightarrow \quad i_2 = -\frac{n_1}{n_2} i_1$$

$$v_1 = \frac{d\psi_1}{dt} = n_1 \frac{d\phi_m}{dt} \qquad v_2 = \frac{d\psi_2}{dt} = n_2 \frac{d\phi_m}{dt} \qquad \Rightarrow v_2 = \frac{n_2}{n_1} v_1$$



Step-down transformer: $v_2 < v_1 \quad \Rightarrow \quad n_2 < n_1 \quad \Rightarrow \quad i_2 > i_1$

coil 2 has fewer turns but higher cross-sectional area

$$v_1\, i_1 = -v_2\, i_2 \ : \quad \text{no losses in the ideal transformer!}$$

Equations of the real transformer

Leakage inductances: $L_{\ell 1} = \dfrac{\psi_{\ell 1}}{i_1}$ $\qquad L_{\ell 2} = \dfrac{\psi_{\ell 2}}{i_2}$

Magnetizing inductance (seen from coil 1) : $L_{m1} = \dfrac{n_1^2}{\mathcal{R}}$

Flux linkages:

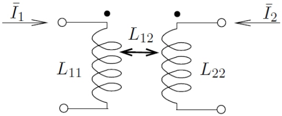
$$\psi_1 = L_{\ell 1}\, i_1 + n_1 \frac{n_1 i_1 + n_2 i_2}{\mathcal{R}} = L_{\ell 1}\, i_1 + \frac{n_1^2}{\mathcal{R}} i_1 + \frac{n_1 n_2}{\mathcal{R}} i_2 = L_{\ell 1}\, i_1 + L_{m1} i_1 + \frac{n_2}{n_1} L_{m1} i_2$$

$$\psi_2 = L_{\ell 2}\, i_2 + n_2 \frac{n_1 i_1 + n_2 i_2}{\mathcal{R}} = L_{\ell 2}\, i_2 + \frac{n_2^2}{\mathcal{R}} i_2 + \frac{n_1 n_2}{\mathcal{R}} i_1 = L_{\ell 2}\, i_2 + (\frac{n_2}{n_1})^2 L_{m1} i_2 + \frac{n_2}{n_1} L_{m1} i_1$$

Voltages at the terminals of the coils:

$$\begin{aligned} v_1 &= R_1 i_1 + \frac{d\psi_1}{dt} = R_1 i_1 + L_{\ell 1} \frac{di_1}{dt} + L_{m1} \frac{di_1}{dt} + \frac{n_2}{n_1} L_{m1} \frac{di_2}{dt} \\ v_2 &= R_2 i_2 + \frac{d\psi_2}{dt} = R_2 i_2 + L_{\ell 2} \frac{di_2}{dt} + (\frac{n_2}{n_1})^2 L_{m1} \frac{di_2}{dt} + \frac{n_2}{n_1} L_{m1} \frac{di_1}{dt} \end{aligned}$$

The transformer is a particular case of magnetically coupled circuits

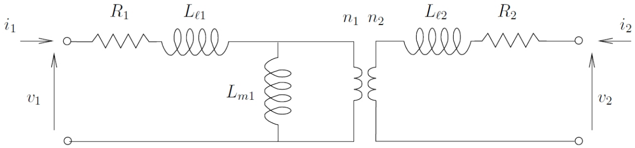


- if the currents enter by the terminals marked with •, their contributions to the flux ϕ_m are added
- if the currents are counted positive when entering the terminals marked with •, the mutual inductance is positive
- the • also indicate AC voltages which are in phase when the transformer is supposed ideal.

One easily identifies:

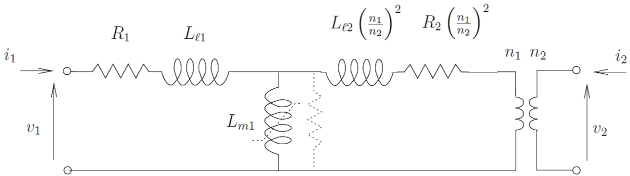
$$L_{11} = L_{\ell 1} + \frac{n_1^2}{\mathcal{R}} \qquad L_{12} = \frac{n_1 n_2}{\mathcal{R}} \qquad L_{22} = L_{\ell 2} + \frac{n_2^2}{\mathcal{R}}$$

Equivalent circuits of the real transformer



$R_1 i_1^2 + R_2 i_2^2$: *copper losses*

Passing R_2 et $L_{\ell 2}$ from side 2 to side 1:



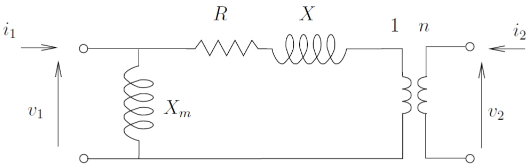
Possible improvements:

- shunt resistance to account for *iron losses* (due to eddy currents)
    - kept small by using laminated cores
    - negligible compared to the power passing through the transformer
- non-linear inductance L_{m1} to account for iron saturation.

Usual simplified equivalent circuit

Usual simplification taking into account that:

$$\omega L_{m1} >> R_1,\ R_2,\ \omega L_{\ell 1},\ \omega L_{\ell 2}$$



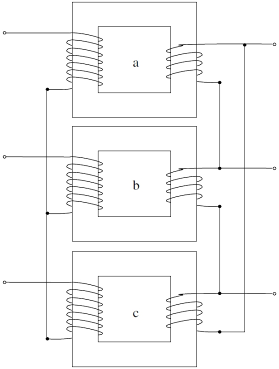
$$n = \frac{n_2}{n_1} \qquad R = R_1 + \frac{R_2}{n^2} \qquad X = \omega L_{\ell 1} + \frac{\omega L_{\ell 2}}{n^2} \qquad X_m = \omega L_{m1}$$

- Equivalent circuit justified by the measurements provided by manufacturers
- X = *leakage reactance* (combined)
- X = *short-circuit reactance* = reactance seen from port 1 when port 2 is short-circuited (considering that X_m is very large compared to X)

The three-phase transformer

First type

Three separate single-phase transformers. No magnetic coupling between phases.

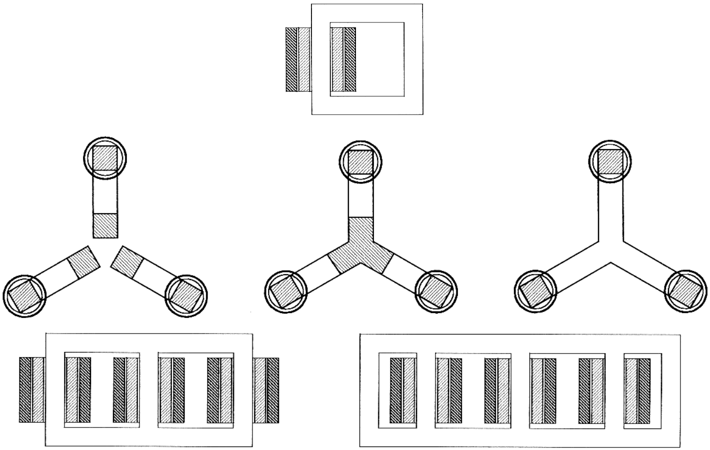


Appropriate for transformers of large nominal power:

- in case of failure of one of the transformers, only that transformer is replaced
- easier to carry.

Second type

The three phases are mounted on a common iron core.



"core" configuration "shell" configuration

Volume of the common core smaller than three times the volume of a single core.

Star vs. Delta configuration

Four possible mountings.

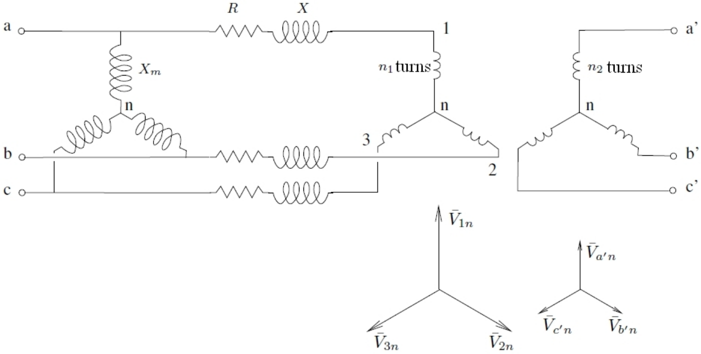
- Transformer side connected to extra high voltage network: star configuration preferred since the voltage across each winding is $\sqrt{3}$ times smaller
- star configuration: possibility to connect the neutral to ground (safety)
- star configuration preferred to place a load tap changer (see later in this chapter)
- transformer side carrying high currents (f.i. the generator side of a step-up transformer): delta configuration preferred since the currents in the branches are $\sqrt{3}$ times smaller
- delta configuration used to eliminate the harmonics of order 3, 6, 9, etc.

Single-phase equivalents of three-phase transformers

- transformer with single core: the phases are magnetically coupled
  → perform a per-phase analysis (see Chapter 2)
- for simplicity, we consider a transformer with three separate cores
- we focus on the impact of the star vs. delta configuration.

1. Star-star configuration

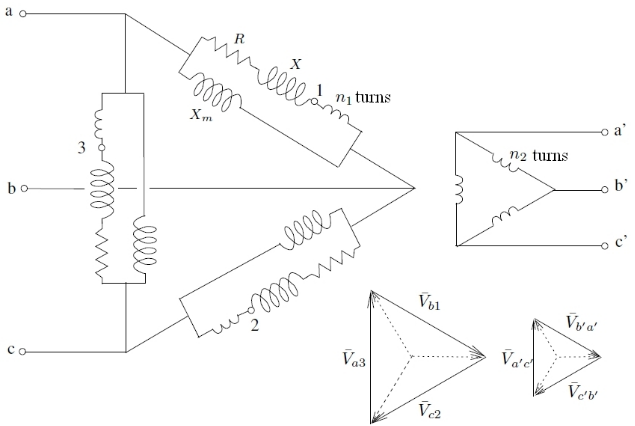
Yy0



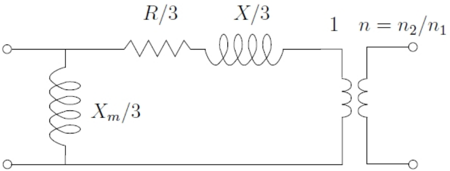
Per-phase equivalent circuit = equivalent circuit of one phase.

2. Delta-delta configuration

Dd0

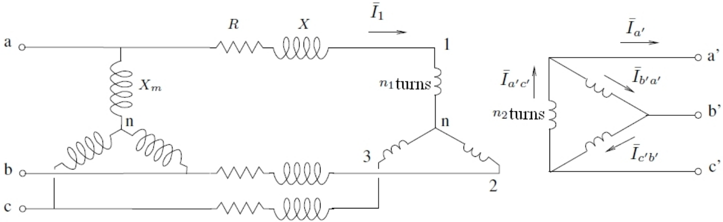


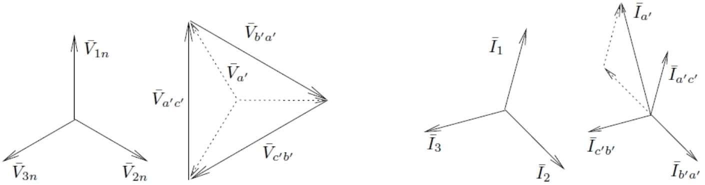
Equivalent circuit:



3. Star-delta configuration

Yd11 / Dy1

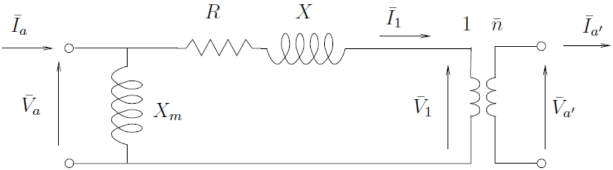




$$\bar{V}_{a'} = \frac{1}{\sqrt{3}}\, e^{j\pi/6}\, \bar{V}_{a'c'} = \frac{n_2}{\sqrt{3}n_1}\, e^{j\pi/6}\, \bar{V}_{1n} = \bar{n}\bar{V}_{1n} \qquad \text{où} \qquad \bar{n} = \frac{n_2}{\sqrt{3}\, n_1}\, e^{j\pi/6}$$

$$\bar{I}_{a'} = \bar{I}_{a'c'} - \bar{I}_{b'a'} = \sqrt{3}\, e^{j\pi/6}\, \bar{I}_{a'c'} = \frac{\sqrt{3}\, n_1}{n_2}\, \frac{1}{e^{-j\pi/6}}\, \bar{I}_1 = \frac{1}{\bar{n}^\star}\, \bar{I}_1$$

Equivalent circuit:



Ideal transformer with complex ratio $\bar{n}$:

- is characterized by : $\bar{V}_{a'} = \bar{n}\bar{V}_1 \qquad \bar{I}_{a'} = \bar{I}_1/\bar{n}^\star$
- reduces to the standard ideal transformer if $\bar{n}$ is real
- transfers complex power without losses: $\bar{V}_{a'}\,\bar{I}^\star_{a'} = \bar{n}\,\bar{V}_1\,\frac{1}{\bar{n}}\,\bar{I}^\star_1 = \bar{V}_1\,\bar{I}^\star_1$

The above two-port is *non reciprocal*: $\bar{I}_a\big]_{\bar{V}_a=0,\bar{V}_{a'}=1} \neq -\bar{I}_{a'}\big]_{\bar{V}_a=1,\bar{V}_{a'}=0}$

4. Delta-star configuration Dy1 / Yd11

Derivation similar to that of the Star-delta configuration,

leading to a single-phase equivalent circuit with:

- the complex transformer ratio: $\bar{n} = \frac{\sqrt{3}\, n_2}{n_1}\, e^{-j\pi/6}$
- a series resistance $R/3$
- a series reactance $X/3$
- a shunt reactance $X_m/3$.

Designation of a transformer

Standardized abbreviation of I.E.C. (International Electrotechnical Commission)

Also referred to as *vector group of a transformer*

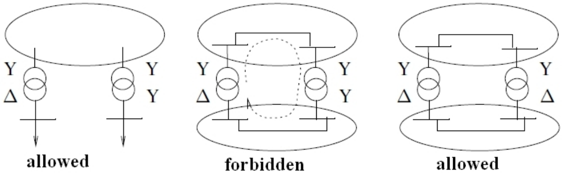
3 symbols:

- an uppercase letter for the high-voltage side: **Y** for a star connection or **D** for a delta
- a lowercase letter for the low-voltage side: **y** for a star connection or **d** for a delta
- an integer $p \in \{0, 1, \ldots, 11\}$:
  - an indication of the phase displacement between the primary and secondary voltages of the same phase, the transformer being assumed ideal
  - the phasor of the high voltage being on the number 12 of a clock, p is the number pointed by the phasor of the low voltage

and for the star configuration:

n after **y** or **Y** to indicate that the neutral is grounded.

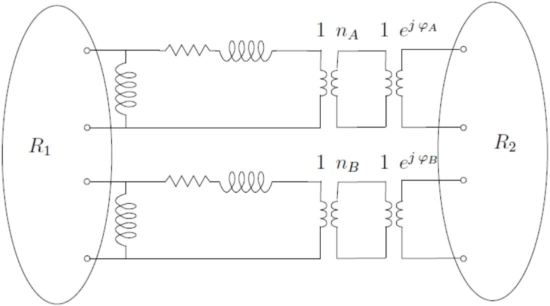
Caution as regards using transformers with different phase displacements



When a given sub-network is fed by two (or more) transformers operating "in parallel" (i.e. located in at least one loop), the latter must have the same phase displacement *p*.

Otherwise, the different phase displacements would cause unacceptable power flows.

Simplification of computations



Two transformers with the same phase displacement: $\varphi_A = \varphi_B = \varphi$

The ideal transformers with complex ratio $e^{j\varphi}$ can be removed without changing:

- the magnitudes of the branch currents and bus voltages
- the complex powers flowing in the branches.

The phase displacements in the transformer models are ignored when computing the steady-state balanced operation of power systems.

Nominal values, per unit system and orders of magnitudes

Nominal values

- Nominal primary voltage U_{1N} and nominal secondary voltage U_{2N}: voltages for which the transformer has been designed (in particular its insulation).
  The real voltages may deviate from these values by a few %.
- Nominal primary current I_{1N} and nominal secondary current I_{2N}: currents for which the transformer has been designed (in particular the cross-sections of the conductors).
  Maximum currents that can be accepted without limit in time.
- nominal apparent power S_N:

$$S_N = \sqrt{3} U_{1N} I_{1N} = \sqrt{3} U_{2N} I_{2N}$$

Conversion of parameters in per unit values

- choose the (three-phase) base power $S_B = S_N$
- on primary side, choose the (phase-to-neutral) base voltage $V_{1B} = U_{1B}/\sqrt{3}$
- on secondary side, choose the (phase-to-neutral) base voltage $V_{2B} = U_{2B}/\sqrt{3}$
- the impedances of the equivalent circuit, which are located on the primary side, are divided by $Z_{1B} = 3V_{1B}^2/S_B = U_{1B}^2/S_B$
- the value of the transformer ratio $n = n_2/n_1$ in per unit is obtained as follows:

$$v_2 = \frac{n_2}{n_1} v_1 \quad \Leftrightarrow \quad v_{2pu} = \frac{v_2}{V_{2B}} = \frac{n_2}{n_1 \; V_{2B}} v_1 = \frac{n_2 \; V_{1B}}{n_1 \; V_{2B}} \frac{v_1}{V_{1B}} = \frac{n_2 \; V_{1B}}{n_1 \; V_{2B}} v_{1pu}$$

$$\Rightarrow \quad n_{pu} = \frac{n_2 \; V_{1B}}{n_1 \; V_{2B}}$$

If $V_{2B}/V_{1B} = n_2/n_1$: $n_{pu} = 1$: the ideal transformer disappears from the equivalent circuit !

In practice, $V_{2B}/V_{1B} \simeq n_2/n_1$: the ideal transformer remains in the equivalent circuit but with a ratio $n_{pu} \simeq 1$.

Orders of magnitude

resistance R	< 0.005 pu
leakage reactance[1] ωL	range: $0.06 - 0.20$ pu
magnetizing reactance ωL_m	range: $20 - 50$ pu
transformer ratio $n = n_2/n_1$	range: $0.85 - 1.15$ pu

values on the (S_B, V_{1B}, V_{2B}) base of the transformer !!

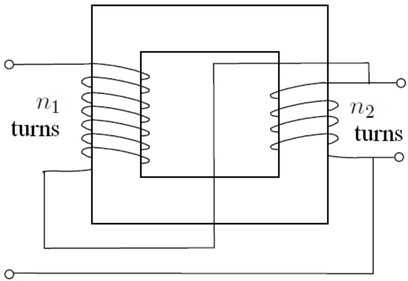
Network computation in another base: convert the parameters to that base (see formula in the chapter on per unit system)

[1] or short-circuit reactance

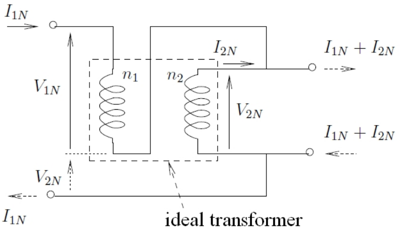
Autotransformers

Single-phase autotransformer

Transformer whose primary and secondary sides are connected in such a way that they have a winding in common:



Let us assume that the inner transformer operates with its voltages and currents at their nominal values (all losses neglected, transformer assumed ideal).



$$
\begin{aligned}
I_{1N}^{auto} &= I_{1N} \qquad\qquad V_{2N}^{auto} = V_{2N} \\
V_{1N}^{auto} &= V_{1N} + V_{2N} = (1 + \frac{n_2}{n_1})V_{1N} \\
I_{2N}^{auto} &= I_{1N} + I_{2N} = (\frac{n_2}{n_1} + 1)I_{2N}
\end{aligned}
$$

Ratio of the autotransformer ?

$$
n^{auto} = \frac{V_{2N}^{auto}}{V_{1N}^{auto}} = \frac{V_{2N}}{V_{1N} + V_{2N}} = \frac{\frac{n_2}{n_1}}{1 + \frac{n_2}{n_1}}
$$

For the chosen primary and secondary, the transformer is of the step-down type.

Nominal apparent power of the autotransformer ?

$$S_N^{auto} = V_{1N}^{auto} I_{1N}^{auto} = (1 + \frac{n_2}{n_1}) V_{1N} I_{1N} = (1 + \frac{n_2}{n_1}) S_N$$

- The autotransformer allows for a power transfer higher than S_N.
  $\Rightarrow$ reduced investment costs and reduced losses !
- True for any n_1, n_2 values but for a higher "amplification": $n_2 \gg n_1$
- However, if $n_2 \gg n_1$, the autotransformer ratio $n^{auto} \to 1$.
  Hence, the device cannot connect two very different voltage levels

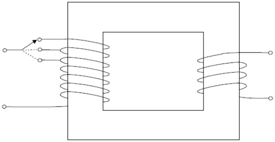
- Autotransformers used to transfer high powers between two networks with relatively close nominal voltages
  - Belgium : 550 MVA autotransformers between 400 and 150 kV
  - France: autotransformers between 400 and 225 kV.
- drawback: metallic connection between primary and secondary $\Rightarrow$ voltage disturbances propagate more easily.

Three-phase autotransformer

Assembly of three single-phase autotransformers.

Adjustment of the turn ratio

Principle



- Objective: adjust voltage at a busbar (usually one of the transformer ends)
- adjustment in steps: between 15 and 25 *tap positions*
- to modify the number of turns in service:
  - transformer taken out of service
  - transformer kept in service: the *on-load* (or *under-load* or *load*) *tap changer* modifies the windings without interrupting the current (avoid electric arcs!)
- load tap changers can be controlled
  - manually: remotely by operator supervising the network from a control center
  - automatically: local feedback system (see chapter on voltage control)
- placement of tap changer:
  - usually on the high-voltage side: current smaller, more turns in winding
  - three-phase transformer: near neutral in Y configuration (lower voltages).

Accounting the tap position changes in equivalent circuit

In principle, one set of $(R, \omega L, \omega L_{m1}, n)$ values for each tap position.

In practice, ωL and n are the most affected, while $R \ll$ and $\omega L_{m1} \gg$.

Possible simplification: let us assume that:

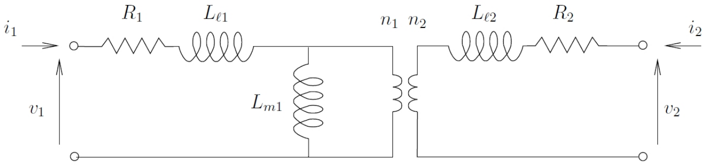
- the turns are adjusted on side 2 in equivalent circuit
- the leakage inductance $L_{\ell 2}$ vary with the number of turns n_2 according to:

$$L_{\ell 2} = L_{\ell 2}^{o} (\frac{n_2}{n_2^{o}})^2$$

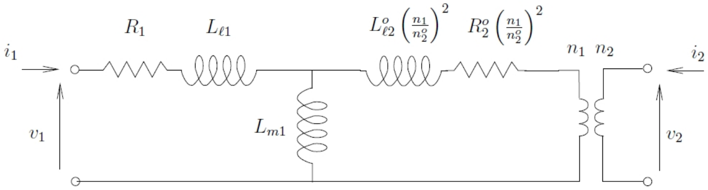
- and similarly for the resistance R_2:

$$R_2 = R_2^{o} (\frac{n_2}{n_2^{o}})^2$$

This is arguable, but R_2 is small...



After passing R_2 and $L_{\ell 2}$ on the other side of the ideal transformer:



When the tap position (and, hence, the number of turns n_2) changes:

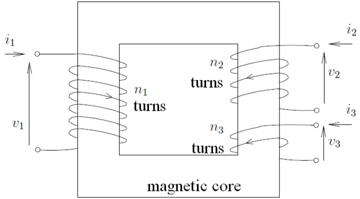
- impedances located on the non-adjusted side remain constant
- only the transformer ratio n_2/n_1 changes.

Three-winding transformers

Shortcut for "transformers with three windings per phase".

Principle

Single-phase transformer with 3 windings (= 1 phase of a 3-phase transformer) :

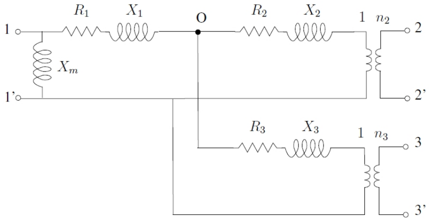


- Power transfer between three voltage levels
- share of power flows between the windings depends on what is connected to the transformer
- nominal apparent powers of the three windings usually different.

Other uses

- in switching stations, power supplied to auxiliaries by the third winding
- connection of a shunt inductance or capacitor for compensation purposes
- improvement of operation in unbalanced condition
- improvement of power quality in the presence of harmonics.

Equivalent circuit



$R_1 + R_2 + j(X_1 + X_2)$: impedance seen from 1 with 2 short-circuited and 3 opened
$R_1 + R_3 + j(X_1 + X_3)$: impedance seen from 1 with 3 short-circuited and 2 opened

Some reactances of this equivalent circuit can be negative
(for instance if the windings have very different nominal apparent powers).

Phase shifting transformer

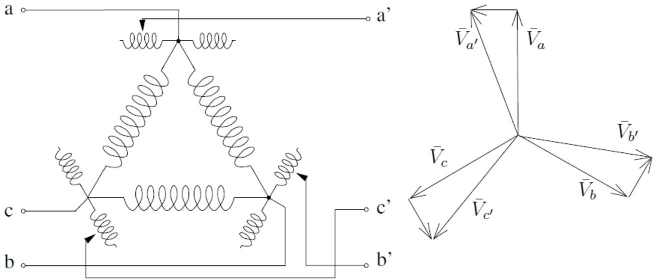
Also called simply *phase shifter*.

Transformer aimed at shifting the secondary voltage phasor with respect to the primary voltage phasor, in order to adjust active power flows in the network.

Two main configurations:

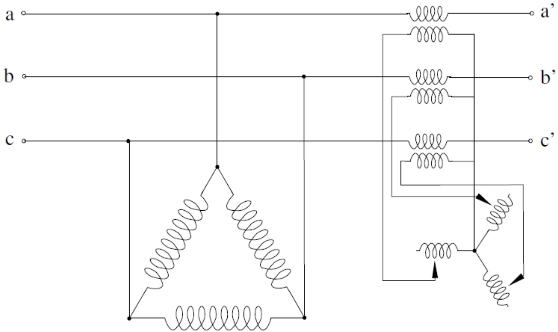
- transformer connecting two networks with different nominal voltages (as usual) to which a device is added to adjust the phase angle
- dedicated device, with the *same* primary and secondary nominal voltages, aimed at adjusting the phase angle.

First scheme



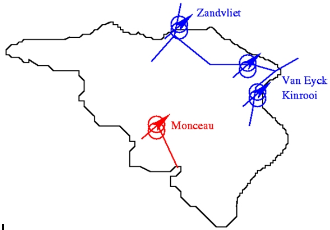
- adjustment *in quadrature*
- some variation of the voltage magnitude with the phase angle
- there exist more elaborate schemes where the voltage magnitude is kept constant while the phase angle is adjusted
- drawback of this scheme: the whole line current passes through the tap changer (unavoidable electric arcs).

Second scheme



- *excitation* shunt transformer + *series* transformer
- nominal voltage of series transformer = fraction of nominal phase-to-neutral voltage $V_N \Rightarrow$ nominal apparent power = fraction of $3 V_N I_{max}$
- compared to previous scheme: lower current in the tap changer.

Example: phase shifting transformers on the borders of Belgium



380/380 kV : in series with:

1. line Zandvliet (B) - Borssele (NL) and Zandvliet (B) - Geertruidenberg (NL)
2. line Meerhout (B) - Maasbracht (NL)
3. line Gramme (B) - Maasbracht (NL)

- nominal power $3V_N I_{max} = 1400$ MVA
- phase shift adjustment: 35 positions, +17/-17 $\times$ 1.5^o (at no load)

220/150 kV : in series with the Chooz (F) - Monceau (B) line

- nominal power: 400 MVA
- in-phase adjustment : 21 positions, +10/-10 $\times$ 1.5 %
- quadrature adjustment: 21 positions, +10/-10 $\times$ 1.2^o