ELEC0014 - Introduction to power and energy systems

The “per unit” system

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Principle

$$\frac{\text{value of a quantity in physical unit}}{\text{value of corresponding “base” in same unit}} = \text{value of quantity in } \textit{per unit (pu)}$$

Advantages:

- the parameters of devices with similar design have close values in per unit, whatever the power of the device, provided they are referred to that power
  - $\Rightarrow$ check of data validity is easier
  - $\Rightarrow$ default values can be substituted to unavailable parameters
- in normal operating conditions, voltages in per unit are close to one
  $\Rightarrow$ better conditioning of numerical computations
- the ideal transformer present in the model of a real transformer disappears after converting the parameters in per unit.

Example

It is known that the internal reactance of a synchronous machine lies typically in the range [1.5 2.5] pu (on the machine base)

- A machine with the characteristics (20 kV, 300 MVA) has a reactance of 2.667 Ω. Is this a normal value ?

  We will see that the base impedance is $20^2/300 = 1.333\ \Omega$
  value of reactance in per unit $= 2.667/1.333 = 2$ *pu*
  $\Rightarrow$ *quite normal value !*

- Same question for a machine with the characteristics (15 kV, 30 MVA)

  The base impedance is now $15^2/30 = 7.5\ \Omega$
  value of reactance in per unit $= 2.667/7.5 = 0.356$ *pu*
  $\Rightarrow$ *abnormal small value !*

Converting a simple circuit in per unit

Converting an electric circuit in per unit $\Longrightarrow$ choosing 3 base quantities

for instance: power S_B voltage V_B time t_B

The other base values are obtained using fundamental laws of Electricity:

base current: $I_B = \dfrac{S_B}{V_B}$

base impedance: $Z_B = \dfrac{V_B}{I_B} = \dfrac{V_B^2}{S_B}$

base magnetic flux: $\psi_B = V_B\, t_B$

base inductance: $L_B = \dfrac{\psi_B}{I_B} = \dfrac{V_B^2\, t_B}{S_B}$

base angular frequency: $\omega_B = \dfrac{Z_B}{L_B} = \dfrac{1}{t_B}$

V_B, I_B: RMS values.

Variant (adopted in this course)

Consider an AC circuit operating at frequency f_N.

Choosing a base angular frequency instead of a base time:

$$\omega_B = \omega_N = 2\pi f_N \quad \text{rad/s}$$

from which one derives:

$$t_B = \frac{1}{\omega_B} = \frac{1}{\omega_N} = \frac{1}{2\pi f_N} \quad \text{s}$$

With this choice:

$$X_{pu} = \frac{X}{Z_B} = \frac{\omega_N\, L}{\omega_B\, L_B} = \frac{L}{L_B} = L_{pu}$$

At frequency f_N, the reactance and the inductance have the same per unit value!

Converting in per unit a typical sinusoidal relation

In MVA, MW and Mvar:

$$S = V\,I\,\cos(\theta - \psi) + j\,V\,I\,\sin(\theta - \psi)$$

In per unit:

$$\begin{aligned} S_{pu} &= \frac{S}{S_B} = \frac{V\,I}{V_B\,I_B}\cos(\theta - \psi) + j\,\frac{V\,I}{V_B\,I_B}\sin(\theta - \psi) \\ &= V_{pu}\,I_{pu}\,\cos(\theta - \psi) + j\,V_{pu}\,I_{pu}\,\sin(\theta - \psi) \end{aligned}$$

Same equation in physical units and in per unit !

In the above steady-state equation, time does not appear explicitly.
Hence, only S_B and V_B are used.

Converting in per unit a typical dynamic equation

In volts:

$$v = R\,i + L\frac{d\,i}{d\,t}$$

In per unit:

$$v_{pu} = \frac{v}{V_B} = \frac{R\,i}{Z_B\,I_B} + \frac{L}{\omega_B\,L_B\,I_B}\frac{d\,i}{d\,t} = R_{pu}i_{pu} + L_{pu}\frac{1}{\omega_B}\frac{d\,i_{pu}}{d\,t} = R_{pu}i_{pu} + L_{pu}\frac{d\,i_{pu}}{d\,t_{pu}}$$

In the above equation, time appears explicitly. There are two options:

- all variables, including time, are converted in per unit
  $\longrightarrow$ identical equations in physical units and in per unit
- time is kept in seconds
  $\longrightarrow$ there appears a factor $1/\omega_B$ in front of the derivation operator.

Converting in per unit two magnetically coupled circuits

Flux-current relations:

$$\begin{aligned} \psi_1 &= L_{11} i_1 + L_{12} i_2 \\ \psi_2 &= L_{21} i_1 + L_{22} i_2 \end{aligned}$$

Identical times. For reasons of simplicity we take: $t_{1B} = t_{2B}$

Symmetry of inductance matrices. In Henry, one has: $L_{12} = L_{21}$. We want that this property still holds true in per unit.

$$\psi_{1pu} = \frac{\psi_1}{\psi_{1B}} = \frac{L_{11}}{L_{1B}} \frac{i_1}{I_{1B}} + \frac{L_{12}}{L_{1B}} \frac{i_2}{I_{1B}} = L_{11pu} i_{1pu} + \underbrace{\frac{L_{12} I_{2B}}{L_{1B} I_{1B}}}_{= L_{12pu}} i_{2pu}$$

Similarly for the second circuit: $$L_{21pu} = \frac{L_{21} I_{1B}}{L_{2B} I_{2B}}$$

$$L_{12pu} = L_{21pu} \quad \Leftrightarrow \quad \frac{I_{2B}}{L_{1B} I_{1B}} = \frac{I_{1B}}{L_{2B} I_{2B}} \quad \Leftrightarrow \quad S_{1B}\, t_{1B} = S_{2B}\, t_{2B} \quad \Leftrightarrow \quad S_{1B} = S_{2B}$$

A per unit system preserving symmetry of inductance matrices is called *reciprocal*.

Summary

circuit # 1	circuit # 2	
S_B	S_B	$\leftarrow$ one degree of freedom
t_B	t_B	$= 1/\omega_N$
V_{1B}	V_{2B}	$\leftarrow$ one degree of freedom in each circuit

Application to a network including several voltage levels connected through transformers:

- the same base power S_B is taken at all voltage levels.
  Usual value in transmission systems: 100 MVA
- the same base time t_B is taken everywhere (not used in steady state)
- at each voltage level, the base voltage is chosen in relation with the nominal voltage of the equipment.

Converting in per unit a three-phase circuit

- Same base time t_B and same base power S_B everywhere
- at each voltage level, a base V_B is chosen for all phase-to-neutral voltages.

1st case. Unbalanced operation - analysis of the three phases

Convenient choice: $S_B =$ single-phase power

$\rightarrow$ base current: $I_B = \dfrac{S_B}{V_B}$

$\rightarrow$ base impedance: $Z_B = \dfrac{V_B}{I_B} = \dfrac{V_B^2}{S_B}$ etc.

Example. Convert in per unit the expression of the three-phase complex power:

$$S = \bar{V}_a \, \bar{I}_a^\star + \bar{V}_b \, \bar{I}_b^\star + \bar{V}_c \, \bar{I}_c^\star$$

$$S_{pu} = \frac{S}{S_B} = \frac{\bar{V}_a}{V_B} \, \frac{\bar{I}_a^\star}{I_B} + \frac{\bar{V}_b}{V_B} \, \frac{\bar{I}_b^\star}{I_B} + \frac{\bar{V}_c}{V_B} \, \frac{\bar{I}_c^\star}{I_B} = \bar{V}_{a\,pu} \bar{I}_{a\,pu}^\star + \bar{V}_{b\,pu} \bar{I}_{b\,pu}^\star + \bar{V}_{c\,pu} \bar{I}_{c\,pu}^\star$$

Identical expressions in physical units and in per unit !

2nd case. Balanced operation - per phase analysis

Convenient choice: S_B = three-phase power

$\rightarrow$ base current: $I_B = \dfrac{S_B}{3V_B} = \dfrac{S_B}{\sqrt{3}U_B}$ $\qquad U_B = \sqrt{3}V_B$

$\rightarrow$ base impedance: $Z_B = \dfrac{V_B}{I_B} = \dfrac{3V_B^2}{S_B} = \dfrac{U_B^2}{S_B}$ $\qquad$ etc.

Example. Convert in per unit the expression of the three-phase complex power:

$$S = \bar{V}_a \, \bar{I}_a^\star + \bar{V}_b \, \bar{I}_b^\star + \bar{V}_c \, \bar{I}_c^\star = 3 \, \bar{V}_a \, \bar{I}_a^\star$$

$$S_{pu} = \frac{S}{S_B} = \frac{3 \, \bar{V}_a \, \bar{I}_a^\star}{3 \, V_B \, I_B} = \bar{V}_{a\,pu} \bar{I}_{a\,pu}^\star$$

1. All calculations are performed in a single phase, and in per unit
2. at the end, the power in all three phases is obtained by multiplying the power in per unit by S_B.

Change of base

In a three-phase circuit, an impedance Z (in ohm) becomes in per unit

in the first base: $$Z_{pu1} = \frac{Z}{Z_{B1}} = \frac{Z\ S_{B1}}{3V_{B1}^2}$$

in the second base: $$Z_{pu2} = \frac{Z}{Z_{B2}} = \frac{Z\ S_{B2}}{3V_{B2}^2}$$

Hence the formula to change from the first to the second base is:

$$Z_{pu2} = Z_{pu1} \frac{S_{B2}}{S_{B1}} \left(\frac{V_{B1}}{V_{B2}} \right)^2$$