ELEC0014 - Introduction to electric power and energy systems

Some properties of electric energy transmission

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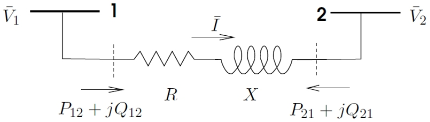
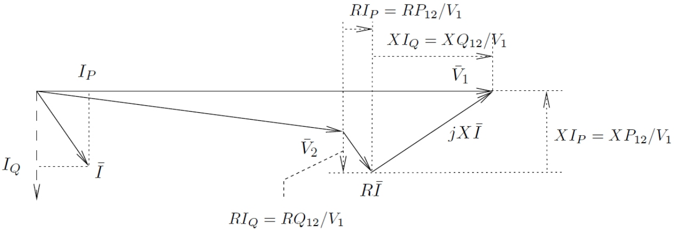
Voltage drop caused by the power flow in an AC link

Model and main relations



$\bar{V}_1 = V_1 \qquad \bar{V}_2 = V_2 e^{j\theta_2} = V_2 \angle \theta_2 \qquad \bar{V}_2 = \bar{V}_1 - (R + jX)\bar{I} \qquad P_{12} + jQ_{12} = \bar{V}_1 \bar{I}^\star$

P_{12}, Q_{12}: powers in per unit

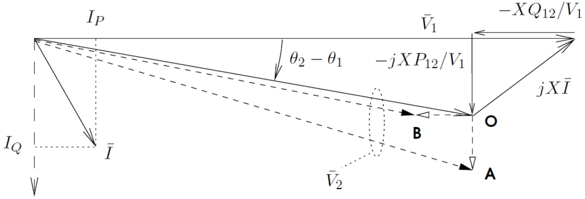


$$\bar{V}_2 = \bar{V}_1 - (R + jX)\frac{P_{12} - jQ_{12}}{V_1} = V_1 - \frac{RP_{12} + XQ_{12}}{V_1} - j\frac{XP_{12} - RQ_{12}}{V_1}$$

Effect of active and reactive power transfers

Transmission network at Extra High Voltage (EHV): R small compared to X.

$$\bar{V}_2 \simeq V_1 - \frac{XQ_{12}}{V_1} - j\frac{XP_{12}}{V_1}$$



Let us assume V_1 is constant and let us consider the following variations:

O $\rightarrow$ A : increase in active power flow

In practice $||\bar{V}_2 - \bar{V}_1|| \ll V_1 \quad \Rightarrow$ *active power flows mainly cause phase shifts of the bus voltage phasors*

O $\rightarrow$ B : increase in reactive power flow

reactive power flows mainly cause variations of the bus voltage magnitudes.

Can reactive power be transmitted over long distances?

In EHV AC transmission networks, reactive power cannot easily be transported over long distances

- complex power balance of the link:

$$P_{12} = -P_{21} + RI^2 \qquad Q_{12} = -Q_{21} + XI^2$$

  since $X >> R$, the reactive losses XI^2 are much larger than the active power losses RI^2. If active and reactive powers enter the link in equal amounts, much less reactive power leaves the link
- large transfers of reactive power $\Rightarrow$ large differences in voltage magnitudes. Not acceptable: voltage magnitudes must not deviate from the nominal values by more than a few percents, for the equipment to work correctly.
- No such limitation for active power since large differences in bus voltage phase angles have no direct consequence for the equipment.

Power transfers as functions of voltages

For the sake of generality, we consider $\theta_1 \neq 0$.

Let us define the admittance:

$$Y = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = |Y|\, e^{j\eta} = |Y|\,(\cos\eta + j\sin\eta)$$

$$P_{12} + jQ_{12} = \bar{V}_1 \bar{I}^\star = \bar{V}_1 \left[Y(\bar{V}_1 - \bar{V}_2) \right]^\star = |Y|\, e^{-j\eta}\, V_1^2 - |Y|\, V_1\, V_2\, e^{j(\theta_1 - \theta_2 - \eta)}$$

$$\Leftrightarrow \quad \begin{aligned} P_{12} &= |Y| \cos\eta\, V_1^2 - |Y|\, V_1\, V_2\, \cos(\theta_1 - \theta_2 - \eta) \\ Q_{12} &= -|Y| \sin\eta\, V_1^2 - |Y|\, V_1\, V_2\, \sin(\theta_1 - \theta_2 - \eta) \end{aligned}$$

A similar derivation (or a simple permutation of indices 1 and 2) yields:

$$\begin{aligned} P_{21} &= |Y| \cos\eta\, V_2^2 - |Y|\, V_1\, V_2\, \cos(\theta_2 - \theta_1 - \eta) \\ Q_{21} &= -|Y| \sin\eta\, V_2^2 - |Y|\, V_1\, V_2\, \sin(\theta_2 - \theta_1 - \eta) \end{aligned}$$

Assuming $R \simeq 0$:

$$|Y| = \frac{1}{X} \qquad \text{and} \qquad \eta = -\frac{\pi}{2}$$

and the above relations become:

$$P_{12} = \frac{V_1 V_2 \sin(\theta_1 - \theta_2)}{X} \tag{1}$$

$$Q_{12} = \frac{V_1^2 - V_1 V_2 \cos(\theta_1 - \theta_2)}{X} \tag{2}$$

$$P_{21} = \frac{V_2 V_1 \sin(\theta_2 - \theta_1)}{X} \tag{3}$$

$$Q_{21} = \frac{V_2^2 - V_2 V_1 \cos(\theta_2 - \theta_1)}{X} \tag{4}$$

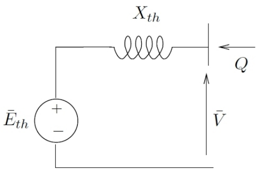
- these relations are useful in many derivations
- but they do not apply to a Medium-Voltage distribution network, where $R \simeq X$!

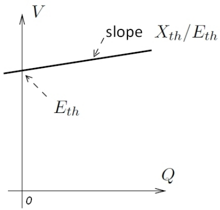
QV characteristic at a busbar of a network

Relation in steady state between the reactive power Q injected in a bus and the voltage V at this bus (any other quantity being unchanged).

Thévenin's theorem. Seen from one port, a linear circuit can be replaced by an equivalent composed of:

- a voltage source = voltage that appears at the port when it is opened
- in series with an impedance = impedance seen from the port after having removed all sources (i.e. voltage sources replaced by short-circuits, current sources by open-circuits)





In our case: E_{th} = voltage at the bus when no reactive power is injected.

Assumption: Thévenin impedance is purely reactive.

Injection of reactive power only $\Rightarrow$ $\bar{E}_{th}$ and $\bar{V}$ are in phase $\Rightarrow$ $Q = \dfrac{V^2 - V\ E_{th}}{X_{th}}$

Linear approximation: $V(Q)$ replaced by a straight line, whose slope is:

$$\frac{1}{\left.\frac{dQ}{dV}\right)_{V=E_{th}}} = \left.\frac{X_{th}}{2V - E_{th}}\right)_{V=E_{th}} = \frac{X_{th}}{E_{th}}$$

Replacing a power system by a Thévenin equivalent entails some approximation:

- not valid for large variations of V and/or Q. Effect of non-linearities:
  - quadratic relation between Q and V
  - switching of generators under reactive power limit $\Rightarrow$ Thévenin parameters change $\Rightarrow$ slope of QV characterisic changes
- after a sudden variation of Q, X_{th} varies with time:

  value in the first instants $\neq$ value in steady state
- following the tripping of a system component (line, transformer, generator), both E_{th} et X_{th} change (quite often E_{th} decreases while X_{th} increases).

Short-circuit capacity

Definition

Also named *fault level* [1].

$$S_{cc} = 3V_N I_{cc} = \sqrt{3} U_N I_{cc}$$

V_N (resp. U_N): nominal phase (resp. line) voltage of the bus of concern
I_{cc}: current in each phase of a zero-impedance three-phase short-circuit at the bus

S_{cc} does not represent a physical power:
V_N = voltage before short-circuit I_{cc} = current during short-circuit.

Important notion, much used in practice.

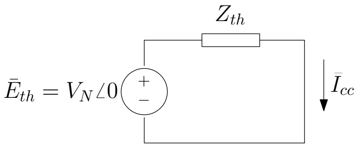
[1] in French: puissance de court-circuit

Relation between short-circuit capacity and Thévenin equivalent

Assume voltage before short-circuit = nominal voltage V_N. Thus: $E_{th} = V_N$

Magnitude of short-circuit current in each phase:

$$|\bar{I}_{cc}| = |\frac{\bar{E}_{th}}{Z_{th}}| = \frac{V_N}{|Z_{th}|}$$



and hence: $$S_{cc} = 3\frac{V_N^2}{|Z_{th}|} = \frac{U_N^2}{|Z_{th}|}$$

Let S_B be the (three-phase) base power. Let us take $U_B = U_N$.

The short-circuit power in per unit :

$$S_{cc,pu} = \frac{S_{cc}}{S_B} = \frac{U_N^2}{|Z_{th}|S_B} = \frac{U_B^2}{S_B}\frac{1}{|Z_{th}|} = \frac{Z_B}{|Z_{th}|} = \frac{1}{|Z_{th,pu}|}$$

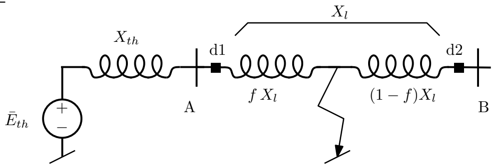
In a transmission system :

$$S_{cc,pu} \simeq \frac{1}{X_{th,pu}}$$

Drawback of too large a short-circuit capacity

For a given voltage level V_N, the higher the short-circuit capacity, the higher the short-circuit currents, which the circuit breakers might not be able to interrupt.

Illustrative example :



Magnitude of current in circuit breaker d1 :

$$I_{cc1} = \frac{E_{th}}{X_{th} + f\,X_l}$$

The larger S_{cc}, the smaller X_{th} and the larger I_{cc1}

Drawbacks of too small a short-circuit capacity

1. When a short-circuit takes place, the nearby voltages fall at low values.
   At the place of the short-circuit this is inevitable.
   At some distance, the voltages should remain high enough in order the loads not be disturbed.

   Previous example : magnitude of voltage at bus A :

   $$V_A = \frac{f\ X_l}{X_{th} + f\ X_l} E_{th}$$

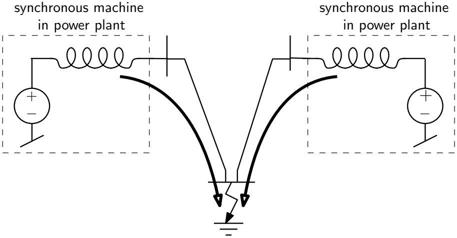
   If f is not too close to zero, the smaller S_{cc}, the larger X_{th} and the lower V_A

2. At a bus with a low S_{cc}, the voltage fluctuates more with the reactive power injected in / withdrawn from this bus.
   $\Rightarrow$ loads with fast-varying reactive power must be connected to buses with a high enough S_{cc}.

3. Some protections are designed to operate with high enough S_{cc}.

Factors influencing the short-circuit capacity (at a given bus)

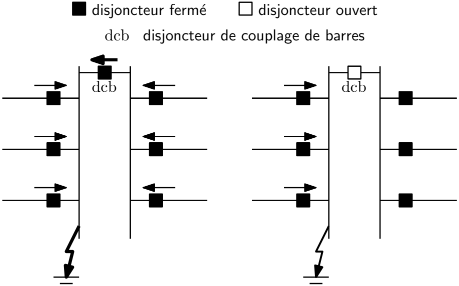
- S_{cc} increases with the number of nearby synchronous machines



- S_{cc} increases with the nominal power of the neighbouring power plants
- problem: replacement of "traditional" power plants by dispersed generation units in distribution networks, interfaced with power electronic converters.

  Those converters do not behave as synchronous machines:
  - no internal e.m.f.
  - protection acting to limit the current (and protect the electronic components)
  - possible disconnection of the unit (by protection reacting to low voltage)

- in low load conditions (during night or in summer), less power plants in operation $\Rightarrow$ lower S_{cc}
- $|Z_{th}|$ decreases (i.e. S_{cc} increases) if there are more lines, cables or transformers connected to the bus of concern (network more meshed)
- this may lead to operate with several, instead of a single bus in a substation



- similarly, the outage of a line connected to a bus decreases the S_{cc} at this bus

- S_{cc} decreases with the nominal voltage U_N of the bus
  - due to the presence of U_N in :

$$S_{cc} = \sqrt{3} U_N I_{cc}$$

  - and because I_{cc} decreases at lower voltage levels, since there are less (powerful) power plants connected at lower voltages.

Orders of magnitude of short-circuit currents and capacities in the Belgian grid

U_N (kV)	I_{cc} (A)	S_{cc} (MVA)
400	45 000	30 000
150	38 000	10 000
70	5 000	600
15	3 000	80

Constraints on power system operation

Very often, constraints force to operate a power system within some limits

- thermal limits : the current in a line, a cable or a transformer must not exceed some threshold
- bus voltages between a minimum and a maximum
- stability limits.

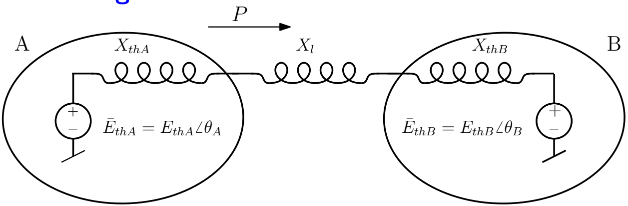
Some constraints are expressed by the maximum power that can be transfered between two locations of a power system.

Remaining of this chapter: maximum power transfer in two basic configurations

- this maximum is intimately linked to AC operation
- beyond the limit, there is no operating point
  - attempting to exceed the limit would result in instability (this requires a dynamic model)
- thermal or voltage constraints are not considered here.

Maximum power transfer between two networks

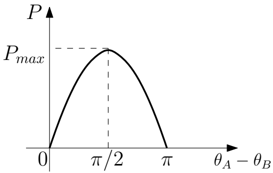
Modelling



$$X_{tot} = X_{thA} + X_l + X_{thB}$$

Power transfer without voltage support in the network

P = active power provided by the e.m.f. $\bar{E}_{thA}$
= active power flow in line
= active power received by the e.m.f. $\bar{E}_{thB}$.



$$P = \frac{E_{thA}E_{thB}}{X_{thA} + X_l + X_{thB}} \sin(\theta_A - \theta_B) = \frac{E_{thA}E_{thB}}{X_{tot}} \sin(\theta_A - \theta_B)$$

Maximum power transfer reached when $\theta_A - \theta_B = \pi/2$ radian :

$$P_{max} = \frac{E_{thA} E_{thB}}{X_{tot}}$$

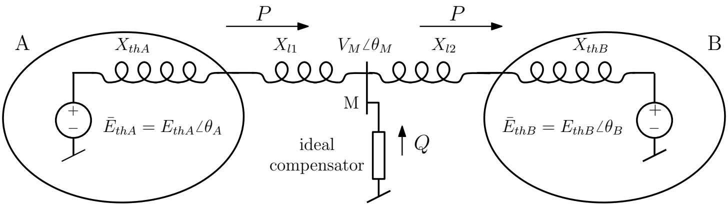
- the larger the reactances X_{thA}, X_{thB} and X_l, the smaller P_{max}
- the larger the voltages E_{thA} and E_{thB}, the larger P_{max}

  $\rightarrow$ advantage of operating power systems at higher voltage !

If, for simplicity, it is assumed that $E_{thA} = E_{thB} = E$:

$$P_{max} = \frac{E^2}{X_{tot}}$$

Power transfer with voltage support in the network



The ideal compensator :

- is designed to keep the V_M voltage constant
- can produce (or consume) reactive power Q
- does not consume or produce active power (losses are neglected)

$$P = \frac{E_{thA}V_M}{X_{thA}+X_{l1}}\sin(\theta_A-\theta_M) = \frac{V_M E_{thB}}{X_{l2}+X_{thB}}\sin(\theta_M-\theta_B)$$

Maximum power transfer reached when either $\theta_A - \theta_M = \pi/2$ or $\theta_M - \theta_B = \pi/2$:

$$P_{max} = \min\left(\frac{E_{thA} V_M}{X_{thA} + X_{l1}}, \frac{V_M E_{thB}}{X_{l2} + X_{thB}}\right)$$

Larger $\theta_A - \theta_B$ than in the case without voltage support !

If, for simplicity, it is assumed that $E_{thA} = V_M = E_{thB} = E$:

$$P_{max} = \min\left(\frac{E^2}{X_{thA} + X_{l1}}, \frac{E^2}{X_{l2} + X_{thB}}\right)$$

The section with the larger reactance is the most constraining.

If, in addition, it is assumed that $X_{thA} + X_{l1} = X_{l2} + X_{thB} = X_{tot}/2$:

$$P_{max} = \frac{2E^2}{X_{tot}} \qquad \text{and} \qquad \theta_A - \theta_M = \theta_M - \theta_B = \pi/2 \tag{5}$$

The maximum power and the phase angle difference are twice as large as in the case without voltage support !

The compensator must be dimensioned to provide the reactive power corresponding to the desired power transfer.

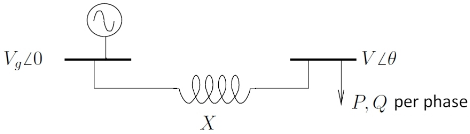
$$Q = \frac{V_M^2 - V_M E_{thA} cos(\theta_M - \theta_A)}{X_{thA} + X_{l1}} + \frac{V_M^2 - V_M E_{thB} cos(\theta_M - \theta_B)}{X_{thB} + X_{l2}}$$

When the power transfer is maximum, in the simplified case of Eq. (5) :

$$Q_{Pmax} = \frac{E^2}{X_{thA} + X_{l1}} + \frac{E^2}{X_{thB} + X_{l2}} = \frac{4E^2}{X_{tot}}$$

Maximum power transfer between a generator and a load

Modelling



From Eqs. (3, 4):

$$
\begin{aligned}
-P &= VV_g\frac{1}{X}\sin\theta \quad \Leftrightarrow \quad P = -\frac{VV_g}{X}\sin\theta \\
-Q &= \frac{1}{X}V^2 - \frac{1}{X}VV_g\cos\theta \quad \Leftrightarrow \quad Q = -\frac{V^2}{X} + \frac{VV_g}{X}\cos\theta
\end{aligned} \tag{6}
$$

We assume that:

- the thermal limit of the line is not reached
- the generator is an ideal voltage source able to provide the requested active and reactive powers.

Conditions of existence of a solution

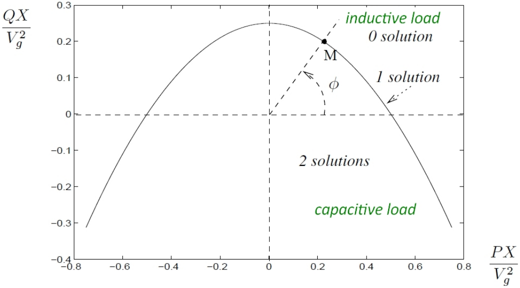
After eliminating θ :

$$\left(V^2\right)^2 + (2QX - V_g^2)V^2 + X^2(P^2 + Q^2) = 0 \quad (7)$$

which is a second-order equation with respect to $y = V^2$.

Condition to have at least one solution:

$$-\left(\frac{P\,X}{V_g^2}\right)^2 - \frac{Q\,X}{V_g^2} + 0.25 \geq 0 \quad (8)$$



$\frac{PX}{V_g^2}$ and $\frac{QX}{V_g^2}$ are dimensionless variables

Interpretation :

- the existence of a maximum power is intimately linked to AC operation
- well known in Circuit theory: extraction of maximum power from a one-port
- but in Circuit theory:
    - the one-port impedance is assumed to have a resistive part: $Z = R + jX$
    - the maximum active power is $V_g^2/4R$ and is obtained by connecting the adapted load impedance $Z^\star = R - jX$, corresponding to capacitive load
    - with $R = 0$ an infinite power could be achieved (as confirmed by the parabola)!

The problem is restated more realistically for a power system as:

find the maximum load power under a given power factor $\cos\phi$

With $Q = P\,\text{tg}\phi$, the maximum load power is reached at point M on the parabola.

Remarks

- for $P < 0$: there is also a maximum power transmissible to the voltage source
- fundamental difference between active and reactive powers:
    - this comes from the fact that the line is reactive, not resistive
    - it is virtually possible to reach any P if enough Q is produced
    - this requires very large voltages V (see Eq. (6)) $\rightarrow$ unacceptable.

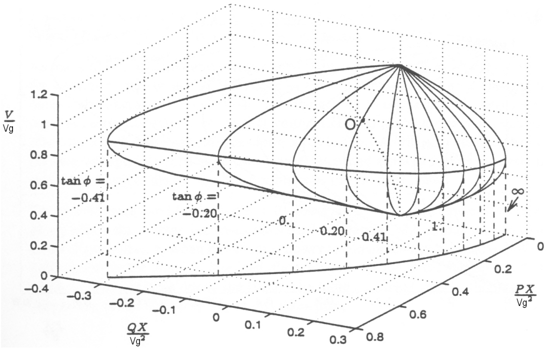
Variation of voltage with load active and reactive powers

Solution of Eq. (7) :

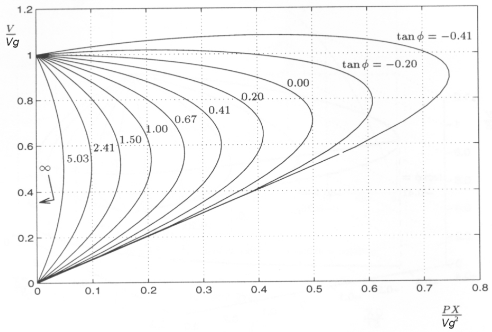
$$y = \frac{V_g^2}{2} - QX \pm \sqrt{\frac{V_g^4}{4} - X^2P^2 - XQV_g^2}$$

Voltage V :

$$V = +\sqrt{y} = \sqrt{\frac{V_g^2}{2} - QX \pm \sqrt{\frac{V_g^4}{4} - X^2P^2 - XQV_g^2}} \tag{9}$$

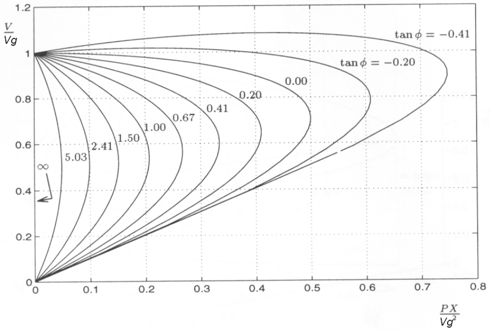


"PV" curves



- one solution (high V, low I) and one solution (low V, high I)
- normal operation is on the upper part of the curves (check: for $P = Q = 0$, $V = V_g$)
- operation on the lower part of the curves is:
  - possible if load was "static"
  - but unstable in the presence of *load tap changers* adjusting the ratios of the transformers feeding distribution networks, to control their voltages (see Chapter 11) = *voltage instability*
- *critical point* corresponding to maximum power = stability limit

"PV" curves



- as the load is more and more compensated (i.e. $tg\phi$ decreases):
  - the maximum active power transmissible to load increases
  - but the voltage at the critical point gets closer to normal value ($V \simeq V_g$)

    $\Rightarrow$ observing normal voltages is no guarantee of large enough security margin
- the same hold true if shunt capacitors are placed in parallel with a constant power factor load
- PV curves are much used in power systems exposed to voltage instability
- real PV curves are computed taking into account the limits on the reactive power produced by generators (non ideal voltage sources).