*ELEC0014 - Introduction to power and energy systems*

# Variation of loads with voltage and frequency

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## Objective

Represent the variation of load power with voltage and frequency

- simple model
- first, considered for individual loads
- next, for groups of loads
- model frequently used in the study of large systems.

# Dynamic vs. static load models

## General dynamic model of load

$$
\begin{aligned}
P &= H_P(V, f, \mathbf{x}) && (1)\\
Q &= H_Q(V, f, \mathbf{x}) && (2)\\
\dot{\mathbf{x}} &= \mathbf{g}(V, f, \mathbf{x})
\end{aligned}
$$

$P$ : active power consumed

$Q$ : reactive power consumed

$V$ : magnitude of terminal voltage

$f$ : frequency of that voltage

$\mathbf{x}$ : state vector characterizing the internal dynamics

## Corresponding static model

Motivation: dynamics very fast, not of interest, or badly known (lack of data)

$$\dot{\mathbf{x}} = \mathbf{0} \quad \Rightarrow \quad \mathbf{g}(V, f, \mathbf{x}) = \mathbf{0} \qquad (3)$$

Obtaining $\mathbf{x}(V, f)$ from (3) and replacing into (1) and (2):

$$P = h_P(V, f) \qquad\qquad Q = h_Q(V, f)$$

# Exponential model

(en français: modèle à exposant(s) )

$$P = P_o \left(\frac{V}{V_o}\right)^{\alpha} \qquad Q = Q_o \left(\frac{V}{V_o}\right)^{\beta} \tag{4}$$

$V_o$ : reference voltage

$P_o$ (resp. $Q_o$) : active (resp. reactive) power consumed under that voltage

$\alpha, \beta$ : characterize the type of load

$P_o, Q_o$ : characterize the amount of load (or connected equipment)

**Power factor** $\cos\phi$

$$\tan\phi = \frac{Q}{P} = \frac{Q_o}{P_o} \left(\frac{V}{V_o}\right)^{\beta-\alpha}$$

if $\alpha \neq \beta$ (which is very often the case in practice) the power factor varies with $V$

# Particular cases

- $\alpha = \beta = 2$ : constant admittance load

$$P = \underbrace{\frac{P_o}{V_o^2}}_{G_o} V^2 \qquad Q = \underbrace{\frac{Q_o}{V_o^2}}_{-B_o} V^2$$

- $\alpha = \beta = 1$ : constant "current"

$$P = \underbrace{\frac{P_o}{V_o}}_{I_{Po}} V \qquad Q = \underbrace{\frac{Q_o}{V_o}}_{I_{Qo}} V$$

Shortcut for "constant active and reactive currents" !
Do not confuse with an independent current source !

- $\alpha = \beta = 0$ : constant power load

$$P = P_o \qquad Q = Q_o$$

## Choice of reference voltage $V_o$

Active power consumed at a voltage $V_1$:

$$P_1 = P_o \left(\frac{V_1}{V_o}\right)^{\alpha} \quad \Rightarrow \quad P_o = P_1 \left(\frac{V_o}{V_1}\right)^{\alpha}$$

By replacing $P_o$ into (4):

$$P = P_1 \left(\frac{V_o}{V_1}\right)^{\alpha} \left(\frac{V}{V_o}\right)^{\alpha} = P_1 \left(\frac{V}{V_1}\right)^{\alpha}$$

Similarly for reactive power:

$$Q = Q_1 \left(\frac{V}{V_1}\right)^{\beta}$$

The reference voltage can be set to any value $V_1$, without changing the load characteristics, provided that the multiplicative constants $P_1$ and $Q_1$ are set to the powers consumed under that voltage.

## Interpretation of the $\alpha$ and $\beta$ exponents

Consider a variation $\Delta V$ of the voltage, small enough to linearize (4) into:

$$\Delta P \;\simeq\; \alpha P_o \frac{V^{\alpha-1}}{V_o^{\alpha}} \Delta V$$

Assume that the variation is around $V = V_o$:

$$\frac{\Delta P}{P_o} \;=\; \alpha \frac{\Delta V}{V_o} \quad \Leftrightarrow \quad \alpha = \frac{\Delta P \,/\, P_o}{\Delta V \,/\, V_o}$$

Similarly for reactive power:

$$\frac{\Delta Q}{Q_o} = \beta \frac{\Delta V}{V_o} \quad \Leftrightarrow \quad \beta = \frac{\Delta Q \,/\, Q_o}{\Delta V \,/\, V_o}$$

$\alpha$ (resp. $\beta$) is the “normalized” or “relative” sensitivity of active (resp. reactive) power to voltage. Dimensionless.

# Accounting for sensitivity to frequency $f$

The frequency variations being small in practice, a linear correction is sufficient:

$$\begin{aligned} P &= P_o \left(1 + D_p \frac{f - f_N}{f_N}\right) \left(\frac{V}{V_o}\right)^{\alpha} \\ Q &= Q_o \left(1 + D_q \frac{f - f_N}{f_N}\right) \left(\frac{V}{V_o}\right)^{\beta} \end{aligned}$$

$f_N$ : nominal frequency of network

The $D_p$ and $D_q$ coefficients are easily interpreted:

$$D_p = \left.\frac{\Delta P \,/\, P_o}{\Delta f \,/\, f_N}\right]_{V=V_o} \qquad \text{et} \qquad D_q = \left.\frac{\Delta Q \,/\, Q_o}{\Delta f \,/\, f_N}\right]_{V=V_o}$$

# Examples of load model parameters

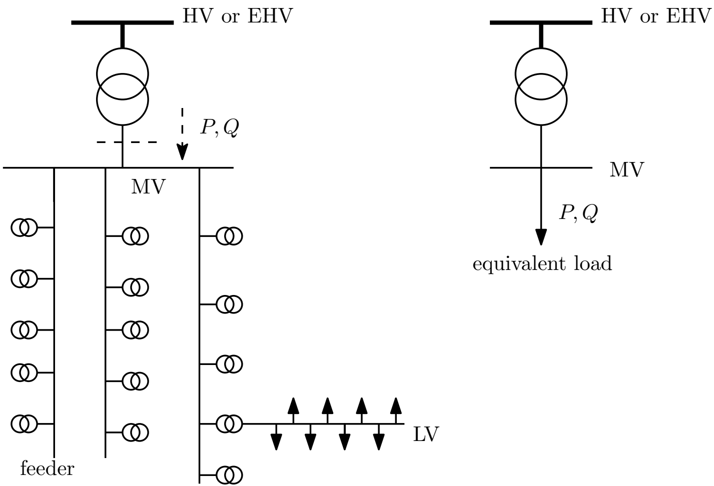
| component | $\cos\phi$ | $\alpha$ | $\beta$ | $D_p$ | $D_q$ |
|---|---|---|---|---|---|
| water heaters, range top, oven, deep fryer | 1.00 | 2.0 | 0 | 0 | 0 |
| dishwasher | 0.99 | 1.8 | 3.6 | 0 | -1.4 |
| clothes washer | 0.65 | 0.08 | 1.6 | 3.0 | 1.8 |
| clothes dryer | 0.99 | 2.0 | 3.2 | 0 | -2.5 |
| refrigerator | 0.8 | 0.77 | 2.5 | 0.53 | -1.5 |
| television | 0.8 | 2.00 | 5.1 | 0 | -4.5 |
| incandescent lights | 1.0 | 1.55 | 0 | 0 | 0 |
| fluorescent lights | 0.9 | 0.96 | 7.4 | 1 | -2.8 |
| industrial motors | 0.88 | 0.07 | 0.5 | 2.5 | 1.2 |
| fan motors | 0.87 | 0.08 | 1.6 | 2.9 | 1.7 |
| agricultural pumps | 0.85 | 1.4 | 1.4 | 5.0 | 4.0 |
| arc furnace | 0.70 | 2.3 | 1.6 | -1.0 | -1.0 |
| unloaded transformer | 0.64 | 3.4 | 11.5 | 0 | -11.8 |
| 3-phase central air conditioning | 0.90 | 0.09 | 2.5 | 0.98 | -1.3 |
| 1-phase central air conditioning | 0.96 | 0.20 | 2.3 | 0.90 | -2.7 |
| window-type air conditioning | 0.82 | 0.47 | 2.5 | 0.56 | -2.8 |

Typical values of $\cos\phi$ allow guessing a reactive power when only the active power of the load is known.

# Model of the whole load of a distribution system

The whole load seen from the entry point of a MV distribution network is a rather complex aggregate

- including numerous loads of different natures
- as well as the losses in the distribution network itself
- load composition is not well known in practice...
- ...and changes with the time of the day, the day of the week, the season, etc.
- even if its composition was well known, there is a need to represent the aggregate with a simple enough model.


HV or EHV
P, Q
MV
feeder
LV
HV or EHV
MV
P, Q
equivalent load

Thanks to its simplicity, the above individual model is frequently used to characterize the power consumed by a set of loads of similar nature

Examples of parameters by load classes

| load class | $\cos\phi$ | $\alpha$ | $\beta$ | $D_p$ | $D_q$ |
|---|---|---|---|---|---|
| residential, in summer | 0.9 | 1.2 | 2.9 | 0.8 | -2.2 |
| residential, in winter | 0.99 | 1.5 | 3.2 | 1.0 | -1.5 |
| commercial, in summer | 0.85 | 1.0 | 3.5 | 1.2 | -1.6 |
| commercial, in winter | 0.9 | 1.3 | 3.1 | 1.5 | -1.1 |
| industrial | 0.85 | 0.2 | 6.0 | 2.6 | 1.6 |
| power plant auxiliaries | 0.8 | 0.1 | 1.6 | 2.9 | 1.8 |

To account for loads of different natures, their respective exponential models can be combined with proportions $a_i$ and $b_i$:

$$P = P_o \left(1 + D_p \frac{f - f_N}{f_N}\right) \frac{\sum_{i=1}^{c} a_i V^{\alpha_i}}{\sum_{i=1}^{c} a_i V_o^{\alpha_i}} \qquad \text{avec} \quad \sum_{i=1}^{c} a_i = 1 \tag{5}$$

$$Q = Q_o \left(1 + D_q \frac{f - f_N}{f_N}\right) \frac{\sum_{i=1}^{c} b_i V^{\beta_i}}{\sum_{i=1}^{c} b_i V_o^{\beta_i}} \qquad \text{avec} \quad \sum_{i=1}^{c} b_i = 1 \tag{6}$$

(same sensitivity to frequency considered for all components of load).

# Limit of validity

Exponential model valid in a certain interval of voltage around the nominal value.

May not accurately apply to large and/or sustained voltage drops, e.g. those created by short-circuits.

Some phenomena responsible for this:

- stalling of induction motors
- disconnection of electronic converters under low voltage conditions
- extinction of fluorescent lights for $V < 0.7$ pu
- etc.