ELEC0014 - Introduction to electric power and energy systems

The overhead power line (and the underground power cable) – Part 2 –

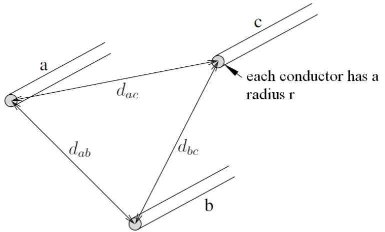
Thierry Van Cutsem

t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

October 2019

Per length unit parameters: series inductance

Simple three-phase line

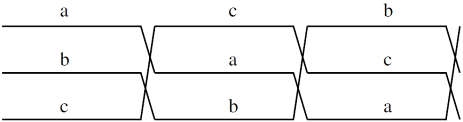


$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \underbrace{\frac{\mu_0}{2\pi} \begin{bmatrix} \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} \\ & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d_{bc}} \\ & & \frac{\mu_r}{4} + \ln\frac{1}{r} \end{bmatrix}}_{\mathbf{L}} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

- derived assuming that $i_a + i_b + i_c = 0$ (no current outside the 3 conductors)
- μ_0: magnetic permeability of vacuum : $\mu_0 = 4\pi 10^{-7}$ H/m
  μ_r: relative magnetic permeability of air / metal of conductors : $\mu_r \simeq 1$
- $\frac{\mu_o \mu_r}{8\pi}$ corresponds to the magnetic field inside each conductor
- $\mathbf{L}$: inductance matrix per length unit. This is a symmetric matrix.

Transposed three-phase line

To cancel the imbalance due to unequal distances between the three conductors.



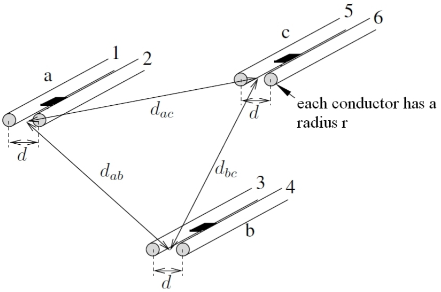
$$\mathbf{L} = \frac{\mu_0}{2\pi}\begin{bmatrix} \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} & \ln\frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & & \frac{\mu_r}{4} + \ln\frac{1}{r} \end{bmatrix}$$

$\sqrt[3]{d_{ab}d_{ac}d_{bc}}$: *Geometric Mean Distance (GMD).*

Per phase per length unit inductance (in H/m) :

$$\ell = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{1}{r} - \ln\frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\right) = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{r}\right)$$

Three-phase line with two conductors per phase

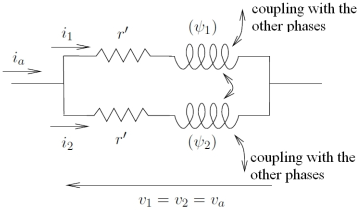


$$d << d_{ab}, d_{ac}, d_{bc}$$

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \psi_5 \\ \psi_6 \end{bmatrix} = \frac{\mu_0}{2\pi} \begin{bmatrix} \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} & \ln\frac{1}{d_{ac}} \\ & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} & \ln\frac{1}{d_{ac}} \\ & & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d} & \ln\frac{1}{d_{bc}} & \ln\frac{1}{d_{bc}} \\ & & & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d_{bc}} & \ln\frac{1}{d_{bc}} \\ & & & & \frac{\mu_r}{4} + \ln\frac{1}{r} & \ln\frac{1}{d} \\ & & & & & \frac{\mu_r}{4} + \ln\frac{1}{r} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

Both conductors of the same phase are:

- kept at the same voltage by the metallic spacers
- identical and at the same distance from the other phase conductors



Thus, it can be assumed: $\psi_1 = \psi_2$ and $v_1 = v_2$

from which one deduces: $i_1 = i_2 = \dfrac{i_a}{2}$

Hence: $$v_a = v_1 = r' \frac{i_a}{2} + \frac{d\psi_1}{dt} = v_2 = r' \frac{i_a}{2} + \frac{d\psi_2}{dt}$$

By writing for phase a: $$v_a = r_a i_a + \frac{d\psi_a}{dt}$$

it is easily identified that: $r_a = \dfrac{r'}{2}$ et $\psi_a = \psi_1 = \psi_2$

By considering every second line in the flux-current relation and adding together the columns relative to equal variables:

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \frac{\mu_0}{2\pi} \begin{bmatrix} \frac{1}{2}\left(\frac{\mu_r}{4} + \ln\frac{1}{d\,r}\right) & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} \\ & \frac{1}{2}\left(\frac{\mu_r}{4} + \ln\frac{1}{d\,r}\right) & \ln\frac{1}{d_{bc}} \\ & & \frac{1}{2}\left(\frac{\mu_r}{4} + \ln\frac{1}{d\,r}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$= \frac{\mu_0}{2\pi} \begin{bmatrix} \left(\frac{\mu_r}{8} + \ln\frac{1}{\sqrt{d\,r}}\right) & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} \\ & \left(\frac{\mu_r}{8} + \ln\frac{1}{\sqrt{d\,r}}\right) & \ln\frac{1}{d_{bc}} \\ & & \left(\frac{\mu_r}{8} + \ln\frac{1}{\sqrt{d\,r}}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$\sqrt{d\,r}$: *Geometric Mean Radius (GMR)*

All other parameters being equal, using a bundle of conductors decreases the self-inductance of each phase ($\frac{\mu_r}{8}$ instead of $\frac{\mu_r}{4}$ and $\sqrt{dr} > r$)

Transposed three-phase line with two conductors per phase

$$\mathbf{L} = \frac{\mu_0}{2\pi} \begin{bmatrix} \left(\frac{\mu_r}{8} + \ln \frac{1}{\sqrt{d\,r}}\right) & \ln \frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} & \ln \frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & \left(\frac{\mu_r}{8} + \ln \frac{1}{\sqrt{d\,r}}\right) & \ln \frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & & \left(\frac{\mu_r}{8} + \ln \frac{1}{\sqrt{d\,r}}\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Per phase per length unit inductance (in H/m) :

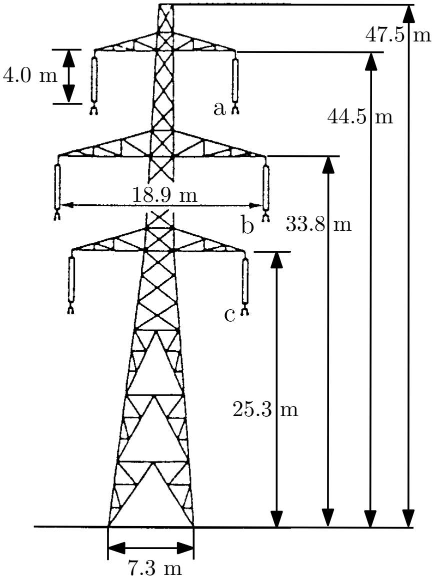
$$\ell = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{8} + \ln \frac{1}{\sqrt{d\,r}} - \ln \frac{1}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\right) = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{8} + \ln \frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{\sqrt{d\,r}}\right)$$

smaller than the inductance of the transposed line with single conductor per phase

How to decrease this equivalent inductance?

- bring the phases closer to each other, all other parameters being equal
- but there must be a minimum insulation distance! The higher the nominal voltage of the line, the higher this distance.

Numerical example



400 kV line

bundle of 2 conductors per phase

$d = 0.4$ m

$r = 0.016$ m

$d_{ab} \simeq 44.5 - 33.8 = 10.7$ m
$d_{bc} \simeq 33.8 - 25.3 = 8.5$ m
$d_{ac} \simeq 44.5 - 25.3 = 19.2$ m

$$H \simeq \frac{(44.5-4)+(33.8-4)+(25.3-4)}{3} = 30.5 \text{ m}$$

Inductance matrix *without* transposition:

$$\mathbf{L} = 10^{-6} \begin{bmatrix} 0.5301 & -0.4740 & -0.5910 \\ -0.4740 & 0.5301 & -0.4280 \\ -0.5910 & -0.4280 & 0.5301 \end{bmatrix} \quad \text{H/m}$$

Inductance matrix *with* transposition:

$$\mathbf{L} = 10^{-6} \begin{bmatrix} 0.5301 & -0.4977 & -0.4977 \\ -0.4977 & 0.5301 & -0.4977 \\ -0.4977 & -0.4977 & 0.5301 \end{bmatrix} \quad \text{H/m}$$

Per phase per km inductance:

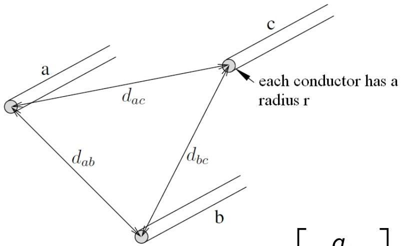
$$\ell = 10^3 10^{-6}(0.5301 + 0.4977) = 1.0278\ 10^{-3} \quad \text{H/km}$$

Per phase per km series reactance:

$$x = 2\pi 50\, \ell = 0.3229 \quad \Omega/\text{km}$$

Per length unit parameters: shunt capacitance

Simple three-phase line



$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \ln\frac{1}{r} & \ln\frac{1}{d_{ab}} & \ln\frac{1}{d_{ac}} & \ln\frac{1}{d_{aa'}} & \ln\frac{1}{d_{ab'}} & \ln\frac{1}{d_{ac'}} \\ \ln\frac{1}{d_{ab}} & \ln\frac{1}{r} & \ln\frac{1}{d_{bc}} & \ln\frac{1}{d_{ba'}} & \ln\frac{1}{d_{bb'}} & \ln\frac{1}{d_{bc'}} \\ \ln\frac{1}{d_{ac}} & \ln\frac{1}{d_{bc}} & \ln\frac{1}{r} & \ln\frac{1}{d_{ca'}} & \ln\frac{1}{d_{cb'}} & \ln\frac{1}{d_{cc'}} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \\ -q_a \\ -q_b \\ -q_c \end{bmatrix}$$

- ϵ (resp. ϵ_0) : permittivity of air (resp. vacuum):

$$\epsilon = \epsilon_0\epsilon_r \simeq \epsilon_0 = \frac{1}{36\pi}10^{-9} \quad \text{F/m}$$

- ground $\equiv$ plane at zero voltage $\rightarrow$ replaced by “image” conductors
- a' = symmetrical with a relative to the ground, at distance $d_{aa'}$ of a, holding a charge $-q_a$. Similarly for phases b and c.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \ln\frac{d_{aa'}}{r} & \ln\frac{d_{ab'}}{d_{ab}} & \ln\frac{d_{ac'}}{d_{ac}} \\ \ln\frac{d_{ba'}}{d_{ab}} & \ln\frac{d_{bb'}}{r} & \ln\frac{d_{bc'}}{d_{bc}} \\ \ln\frac{d_{ca'}}{d_{ac}} & \ln\frac{d_{cb'}}{d_{bc}} & \ln\frac{d_{cc'}}{r} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

Since the conductors are at relatively high distance from the ground:

$$d_{aa'} \simeq d_{ab'} \simeq d_{ac'} \simeq d_{ba'} \simeq d_{bb'} \simeq d_{bc'} \simeq d_{ca'} \simeq d_{cb'} \simeq d_{cc'} \simeq 2H \qquad (1)$$

H : average height of conductors above ground.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \underbrace{\frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \ln\frac{2H}{r} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ac}} \\ & \ln\frac{2H}{r} & \ln\frac{2H}{d_{bc}} \\ & & \ln\frac{2H}{r} \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

Transposed three-phase line

$$\mathbf{S} = \frac{1}{2\pi\epsilon_o\epsilon_r}\begin{bmatrix} \ln\frac{2H}{r} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & \ln\frac{2H}{r} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & & \ln\frac{2H}{r} \end{bmatrix}$$

Per phase per length unit capacitance c [1] ?

We assume: $q_a + q_b + q_c = 0$

$$\begin{aligned} v_a &= \frac{1}{2\pi\epsilon_o\epsilon_r}\left(\ln\frac{2H}{r}\, q_a + \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\,(q_b+q_c)\right) \\ &= \frac{1}{2\pi\epsilon_o\epsilon_r}\left(\ln\frac{2H}{r} - \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\right) q_a \end{aligned}$$

$$\Rightarrow \quad c = 2\pi\epsilon_o\epsilon_r \frac{1}{\ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{r}} \quad \text{(F/m)}$$

Does not depend of H. (1) amounts to neglecting the influence of ground.

[1]this is the capacitance $C + 3C_m$ in Fig. 7.b of "Balanced three-phase systems and operation"

Three-phase line with two conductors per phase

By making the same approximations on the distances between conductors:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \ln\frac{2H}{r} & \ln\frac{2H}{d} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ac}} & \ln\frac{2H}{d_{ac}} \\ & \ln\frac{2H}{r} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ac}} & \ln\frac{2H}{d_{ac}} \\ & & \ln\frac{2H}{r} & \ln\frac{2H}{d} & \ln\frac{2H}{d_{bc}} & \ln\frac{2H}{d_{bc}} \\ & & & \ln\frac{2H}{r} & \ln\frac{2H}{d_{bc}} & \ln\frac{2H}{d_{bc}} \\ & & & & \ln\frac{2H}{r} & \ln\frac{2H}{d} \\ & & & & & \ln\frac{2H}{r} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$

Equal distribution of charges:

$$q_1 = q_2 = \frac{q_a}{2} \qquad q_3 = q_4 = \frac{q_b}{2} \qquad q_5 = q_6 = \frac{q_c}{2}$$

Conductors at the same voltage:

$$v_1 = v_2 = v_a \qquad v_3 = v_4 = v_b \qquad v_5 = v_6 = v_c$$

By considering every second line in the voltage-charge relation and adding together the columns relative to equal variables:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \frac{1}{2}\left(\ln\frac{4H^2}{d\,r}\right) & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ac}} \\ & \frac{1}{2}\left(\ln\frac{4H^2}{d\,r}\right) & \ln\frac{2H}{d_{bc}} \\ & & \frac{1}{2}\left(\ln\frac{4H^2}{d\,r}\right) \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

$$= \frac{1}{2\pi\epsilon_o\epsilon_r} \begin{bmatrix} \ln\frac{2H}{\sqrt{d\,r}} & \ln\frac{2H}{d_{ab}} & \ln\frac{2H}{d_{ac}} \\ & \ln\frac{2H}{\sqrt{d\,r}} & \ln\frac{2H}{d_{bc}} \\ & & \ln\frac{2H}{\sqrt{d\,r}} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

Transposed three-phase line with two conductors per phase

$$\mathbf{S} = \frac{1}{2\pi\epsilon_o\epsilon_r}\begin{bmatrix} \ln\frac{2H}{\sqrt{d\,r}} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & \ln\frac{2H}{\sqrt{d\,r}} & \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}} \\ & & \ln\frac{2H}{\sqrt{d\,r}} \end{bmatrix}$$

Per phase per length unit capacitance c ?

We assume again: $q_a + q_b + q_c = 0$

$$\begin{aligned} v_a &= \frac{1}{2\pi\epsilon_o\epsilon_r}\left(\ln\frac{2H}{\sqrt{d\,r}}\,q_a + \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\,(q_b+q_c)\right) \\ &= \frac{1}{2\pi\epsilon_o\epsilon_r}\left(\ln\frac{2H}{\sqrt{d\,r}} - \ln\frac{2H}{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}\right) q_a \end{aligned}$$

$$\Rightarrow \quad c = 2\pi\epsilon_o\epsilon_r\frac{1}{\ln\frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{\sqrt{d\,r}}}$$

Here again, the result is independent of H.

Numerical example (continued)

S matrix *without* transposition:

$$\mathbf{S} = 10^{11} \begin{bmatrix} 1.1946 & 0.3133 & 0.2081 \\ 0.3133 & 1.1946 & 0.3547 \\ 0.2081 & 0.3547 & 1.1946 \end{bmatrix} \quad \text{m/F}$$

S matrix *with* transposition:

$$\mathbf{S} = 10^{11} \begin{bmatrix} 1.1946 & 0.2920 & 0.2920 \\ 0.2920 & 1.1946 & 0.2920 \\ 0.2920 & 0.2920 & 1.1946 \end{bmatrix} \quad \text{m/F}$$

Capacitance matrix:

$$\mathbf{C} = \mathbf{S}^{-1} = 10^{-11} \begin{bmatrix} 0.9261 & -0.1819 & -0.1819 \\ -0.1819 & 0.9261 & -0.1819 \\ -0.1819 & -0.1819 & 0.9261 \end{bmatrix} \quad \text{F/m}$$

Per phase per km capacitance: this is an alternative way of computing

$$c = 10^3 \; 10^{-11} \; (0.9261 + 0.1819) = 1.108 \; 10^{-8} \quad \text{F/km}$$

Per phase per km shunt susceptance:

$$b = 2\pi 50 \; c = 3.48 \; 10^{-6} \;\; \text{S/km} \;\; = 3.48 \;\; \mu\text{S/km}$$

Per length unit parameters: typical values

Sample of HV and EHV lines from the Belgian grid

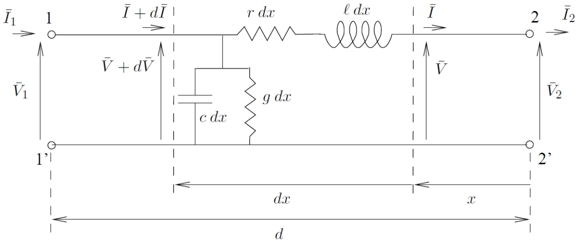
	nominal voltages (kV)			
	380	220	150	70
r (Ω/km)	0.03	0.04 – 0.09	0.05 – 0.12	0.09 – 0.35
$\omega\ell$ (Ω/km)	0.3 (2)	0.3 (2) ou 0.4 (1)	0.4 (1)	0.2 – 0.4 (1)
ωc (μS/km)	3.0	3.0	3.0	3.0
S_{max} (MVA)	1350 ou 1420	250–500	150 – 350	30 – 100

(1) 1 conductor per phase (2) 2 conductors per phase

What about the shunt conductance?

- very small
- caused by leakage currents at the surface of insulator strings
  - in dusty industrial environment
  - due to salty atmosphere near the sea
- corresponding Joule losses negligible compared to power through the line.

The line as a distributed component



per length unit series impedance: $z = r + j\omega\ell \quad (\Omega/m)$
per length unit shunt admittance: $y = g + j\omega c \quad (S/m)$

Ohm's and Kirchhoff's laws in the section of infinitesimal length dx :

$$d\bar{V} = zdx\ \bar{I} \quad \Rightarrow \quad \frac{d\bar{V}}{dx} = z\bar{I}$$

$$d\bar{I} = ydx\ (\bar{V} + d\bar{V}) \simeq ydx\ \bar{V} \quad \Rightarrow \quad \frac{d\bar{I}}{dx} = y\bar{V}$$

$$\Leftrightarrow \quad \frac{d^2\bar{V}}{dx^2} = yz\bar{V} = \gamma^2\bar{V} \qquad \gamma = \sqrt{yz} : \textit{propagation constant}(m^{-1})$$

General solution of the diff. equ.

$$\bar{V} = k_1\ e^{\gamma x} + k_2\ e^{-\gamma x}$$

Interpretation

$$k_1 = |k_1| e^{j\nu_1} \qquad k_2 = |k_2| e^{j\nu_2} \qquad \gamma = \alpha + j\,\beta$$

$$\bar{V} = |k_1| e^{\alpha x} e^{j(\beta x + \nu_1)} + |k_2| e^{-\alpha x} e^{j(-\beta x + \nu_2)}$$

Voltage at time t and at the coordinate x:

$$v(x,t) = \underbrace{\sqrt{2}|k_1|e^{\alpha x}\cos(\omega t + \beta x + \nu_1)}_{v_1(x,t)} + \underbrace{\sqrt{2}|k_2|e^{-\alpha x}\cos(\omega t - \beta x + \nu_2)}_{v_2(x,t)}$$

$v_1(x,t)$: wave propagating with attenuation from left to right = *incident wave*

α: *attenuation constant* $\qquad$ β: *phase constant*

$v_2(x,t)$: wave propagating with attenuation from right to left = *reflected wave*

Velocity of propagation $v = \dfrac{\omega}{\beta}$ = speed of light in air $\simeq$ 300 000 km/s

Wavelength $\lambda = \dfrac{2\pi}{\beta} = \dfrac{v}{f} \simeq$ 6.000 km at 50 Hz $\gg$ length of the longest lines

Relation between voltages and currents at one point and at the line end

$$\bar{V} = k_1\, e^{\gamma x} + k_2\, e^{-\gamma x} = (k_1 + k_2)\,\frac{e^{\gamma x} + e^{-\gamma x}}{2} + (k_1 - k_2)\,\frac{e^{\gamma x} - e^{-\gamma x}}{2}$$
$$= K_1 \operatorname{ch} \gamma x + K_2 \operatorname{sh} \gamma x \qquad (2)$$

At $x = 0$ we have:

- $\bar{V} = \bar{V}_2 \quad \Rightarrow \quad K_1 = \bar{V}_2$
- $\bar{I} = \bar{I}_2 \quad \Rightarrow \quad \left.\frac{d\bar{V}}{dx}\right]_{x=0} = z\bar{I}_2 \quad \Rightarrow \quad K_1\gamma \operatorname{sh} \gamma x + K_2\gamma \operatorname{ch} \gamma x\big]_{x=0} = z\bar{I}_2$

$$\Rightarrow \quad K_2 = \frac{z\bar{I}_2}{\gamma} = \sqrt{\frac{z}{y}}\,\bar{I}_2 = Z_c \bar{I}_2$$

$Z_c = \sqrt{\frac{z}{y}}$: *characteristic impedance (or surge impedance)* (Ω)

Replacing K_1 and K_2 in (2), and following the same procedure for $\bar{I}$:

$$\bar{V} = \bar{V}_2 \operatorname{ch} \gamma x + Z_c \bar{I}_2 \operatorname{sh} \gamma x \qquad (3)$$

$$\bar{I} = \frac{\bar{V}_2}{Z_c} \operatorname{sh} \gamma x + \bar{I}_2 \operatorname{ch} \gamma x \qquad (4)$$

A few properties of characteristic impedance

Approximation: lossless line: $r = 0 \quad g = 0 \quad \Rightarrow \quad z = j\omega\ell \quad y = j\omega c$

$$\gamma = j\beta = j\omega\sqrt{\ell c} \qquad Z_c = |Z_c| = \sqrt{\frac{\ell}{c}} \quad \text{: this is a resistance}$$

$$\bar{V} = \bar{V}_2 \cos\beta x + jZ_c\bar{I}_2 \sin\beta x \qquad \bar{I} = \bar{I}_2 \cos\beta x + j\frac{\bar{V}_2}{Z_c} \sin\beta x$$

When the line is closed on its impedance Z_c :

$$\bar{V}_2 = Z_c\bar{I}_2 \quad \Rightarrow \quad \bar{V} = \bar{V}_2\, e^{j\beta x} \quad \bar{I} = \bar{I}_2\, e^{j\beta x}$$

- no reflected wave!
- voltage and current of constant RMS value everywhere along the line
- voltage and current phasors are in phase everywhere along the line
- impedance $\bar{V}/\bar{I}$ seen from any point $= Z_c$
- the line neither consumes nor produces reactive power: $\omega c V^2 = \omega\ell I^2$
- three-phase active power flowing at any point of the line: $P_c = 3V_2^2/Z_c$
- these properties hold true whatever the length of the line!

Natural power (or Surge Impedance Loading - SIL)

Active power flowing at any point of a lossless line (or cable) when it is closed on its characteristic impedance Z_c and operates under its nominal voltage

$$P_c = \frac{3V_N^2}{Z_c} = \frac{U_N^2}{Z_c}$$

V_N: nominal phase-to-neutral voltage U_N: nominal line voltage

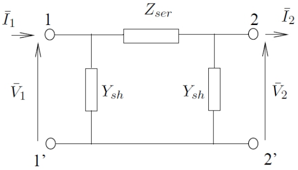
- cables used to transmit data are usually closed on their characteristic impedance to avoid wave reflexion
- this is not the case for a transmission line, which rarely ("by chance") operates at its natural power
- natural power is merely a "reference" operating point characterized by a zero reactive power balance.

Pi-equivalent two-port of a line

Consider Eqs. (3, 4) for $x = d$:

$$\bar{V}_1 = \bar{V}_2 \text{ ch } \gamma d + Z_c \bar{I}_2 \text{ sh } \gamma d \quad (5)$$

$$\bar{I}_1 = \frac{\bar{V}_2}{Z_c} \text{ sh } \gamma d + \bar{I}_2 \text{ ch } \gamma d \quad (6)$$



Values of Z_{ser} and Y_{sh} so that the pi-equivalent with lumped components and the distributed line have the same behaviour *seen from the rest of the network* ?

$$\bar{V}_1 = \bar{V}_2 + Z_{ser}(\bar{I}_2 + Y_{sh}\bar{V}_2) = (1 + Z_{ser}Y_{sh})\bar{V}_2 + Z_{ser}\bar{I}_2$$

$$Z_{ser} = Z_c \text{ sh } \gamma d$$

$$1 + Z_{ser}Y_{sh} = \text{ch } \gamma d \quad \Leftrightarrow \quad Y_{sh} = \frac{\text{ch } \gamma d - 1}{Z_c \text{ sh } \gamma d} = \frac{1}{Z_c} \text{th} \frac{\gamma d}{2}$$

Case of a “short” line

$|\gamma d|$ small enough to limit the Taylor series expansion to the first term:

$$\begin{aligned} \text{sh}\,\gamma d &\simeq \gamma d + \ldots \\ \text{th}\,\frac{\gamma d}{2} &\simeq \frac{\gamma d}{2} + \ldots \end{aligned}$$

Parameters of the pi-equivalent:

$$\begin{aligned} Z_{ser} &= Z_c \gamma d = \sqrt{\frac{z}{y}}\sqrt{zy}\,d = zd \\ Y_{sh} &= \frac{1}{Z_c}\frac{\gamma d}{2} = \frac{1}{2}\sqrt{\frac{y}{z}}\sqrt{zy}\,d = \frac{1}{2}yd \end{aligned}$$

parameters of pi-equivalent = per length unit values × length of line

In practice, this is accurate enough for a line length smaller than $\simeq$ 150 km.

Thermal limit (or capacity) of a line or cable

The thermal limit (or capacity) of a line or a cable is usually specified by its admissible nominal apparent power:

$$S_{max} = 3V_N I_{max} = \sqrt{3} U_N I_{max}$$

I_{max} : maximum admissible current in one phase
V_N : nominal phase-to-neutral voltage
U_N : nominal phase-to-phase voltage

Examples

380-kV overhead line thermal limit : 1350 MVA

- aluminum with steel core
- bundle of two conductors; section of each conductor: 620 mm^2

$$I_{max} = \frac{1350\ 10^6}{\sqrt{3}\ 380\ 10^3} = 2051 \ \ \text{A/phase} \longrightarrow 1025 \ \text{A/conductor}$$

- current density : $\frac{1025}{620} = 1.65 \ \text{A/mm}^2$

36-kV underground cable thermal limit : 25 MVA

- section of each phase: 400 mm^2

$$I_{max} = \frac{25\ 10^6}{\sqrt{3}\ 36\ 10^3} = 401 \ \ \text{A/phase}$$

- current density : $\frac{401}{400} = 1.0 \ \text{A/mm}^2$

Power cables: description

Usage

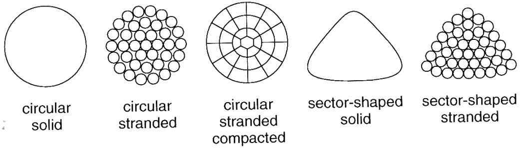
- power transmission and distribution in densely populated areas
- submarine power transmission
- visual impact: preferred to overhead lines for new connections, or when replacing old lines
- little exposed to bad weather conditions ⇒ more reliable than overhead lines
- but 6 to 10 times more expensive than overhead line of same power rating
- more complex maintenance (no visual inspection, necessity to dig).

Main design constraints

- The cable must remain flexible enough to fit on a drum
- the cable operates in a confined space
    - heating may damage the insulating material
    - the cable is not cooled by ambient air as the overhead line
- the conductors must be insulated
    - not insulated by ambient air as the overhead line.

Conductor material

- The higher weight of copper is not an issue
- the lower resistivity of copper compared to aluminum yields lower Joule losses
- copper is preferred but aluminum is also used.



source: [2]

Three-core vs. single core cables (see pictures)

- Higher voltage ⇒ thicker conductors
  - to reduce the electric field strength at the conductor surface
- higher voltage ⇒ voluminous insulating material
- for voltage higher than ≃ 60 kV, three single-core cables preferred to one three-core cable
- two types of three-core cables:
  - three phase conductors inside a single sheath (*belted cable*)
  - each of the three phase conductors has its own sheath.

Three-core cable

three phase conductors inside a single sheath

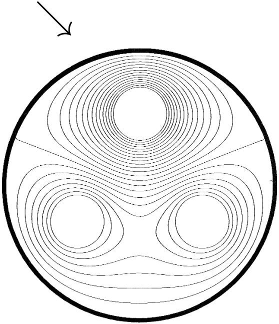
20 kV

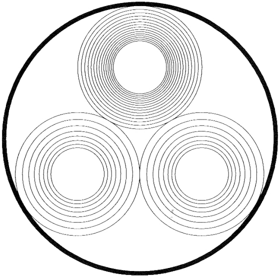
cross section area: 240 mm^2

"snapshot" of the equipotential lines in a three-core cable

upper phase at maximum positive voltage; lower two phases at half the minimum negative value

source: [2]





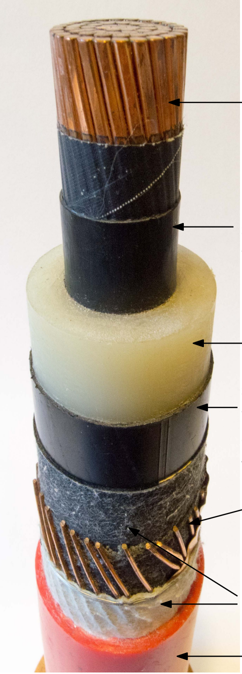
"snapshot" of the equipotential lines when each of the three phase conductors has its own sheath

upper phase at maximum positive voltage; lower two phases at half the minimum negative value

Single-core cable

36 kV 25 MVA

cross section area: 400 mm^2



stranded conductor

semiconducting shield to: (i) distribute the field uniformly around the conductor, and (ii) prevent ionized voids

insulation (XLPE)

semiconducting shield to: (i) reduce the surface voltage to zero, and (ii) confine the electric field to the insulation (eliminating tangential stress)

copper (lattice) sheath, grounded, serves as neutral (carries current in case of phase imbalance, due to e.g. a fault)

absorbing paper to avoid propagation of humidity in case the cable is damaged locally

mechanical protection

Cross section of a single-core cable with aluminum conductor

Insulating material

- Paper impregnated with oil (to fill the spaces in the cellulose structure)
  - has been used quite a lot, excellent insulation properties
- variant: oil-filled cable
  - conductor with hollow center filled with oil under moderate pressure maintained by reservoirs feeding the cable along the route
  - when cable warms up, oil expands and is driven from cable into reservoirs
  - gaps in insulating material are avoided, no weak point present.
  - danger of pollution by oil !
- plastic insulation is preferred nowadays
  - polymers with specific chemical, electrical and mechanical properties
  - many types exist. The most popular are: *PolyEthylene (PE)* and *cross-Linked PolyEthylene (XLPE)*
  - manufacturing: plastic melted and pressed around the conductor
  - insulation must be free of cavities and inclusions (dust, fibers, metal particles, etc.) to prevent partial discharges (same as corona effect around overhead line)
  - vulnerable to water and water vapor (lower dielectric withstand level).

Metallic shields

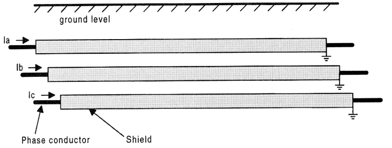
- Purposes: electrostatic sheath, return/neutral wire, protection against water.

Grounding of metallic shields (single-core cables)

Eddy currents are induced in the metallic shield of each cable by the current in its center conductor as well as by the currents in the adjacent cables.

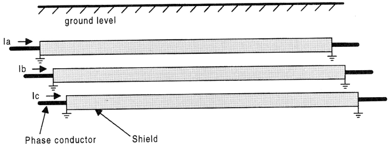
Shields grounded at a single end

A voltage is induced along the shield, proportional to the cable length.
This voltage could reach high values.
Acceptable for short distances only.



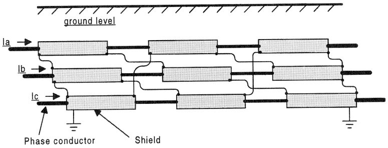
Shields grounded at both ends

A path exist for currents to flow through the shields.
It is used as return wire in case of imbalance due to e.g. a fault.
But these currents causes losses and heat, which may reduce the thermal capacity of the cable.



Shields grounded at both ends, sectionalized and cross-connected

The total induced voltage in the consecutive sections is almost neutralized. No heat is generated. Longer route lengths can be realized.



Per length unit parameters of cables

Series reactance

The derivations made for the overhead lines can be applied to the cable (metallic shields have negligible influence on the *magnetic* field).

$$\ell = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln \frac{\sqrt[3]{d_{ab}d_{ac}d_{bc}}}{r}\right)$$

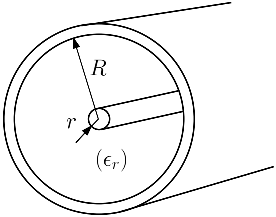
Phases closer to each other $\Rightarrow \omega\ell$ smaller in cables.

Shunt susceptance. We only consider a single-core cable (or a three-core one, in which each phase conductor has its own sheath).

Per length unit capacitance of a capacitor made up of two co-axial cylinders

$$c = \frac{2\pi\epsilon_o\epsilon_r}{\ln\left(\frac{R}{r}\right)}$$

R much smaller than its equivalent in overhead line
$\epsilon_r \simeq 2.3$ for XPLE, 3.5 for paper-oil (1 for air)
$\Rightarrow \omega c$ (much) higher in cables.



Average values from a sample of cables in the Belgian grid

	nominal voltages (kV)	
	150	36
r (Ω/km)	0.03 – 0.12	0.06 – 0.16
$\omega\ell$ (Ω/km)	0.12 – 0.22	0.10 – 0.17
ωc (μS/km)	30 – 70	40 – 80
S_{max} (MVA)	100 – 300	10 – 30

High shunt susceptance prevents using HV or EHV cables over long distances:

- total capacitive current becomes prohibitively high $\Rightarrow$ net power that can be transmitted is significantly reduced
- produces a lot of reactive power $\Rightarrow$ overvoltage problems
- compensation by shunt inductors needed along the cable
  - increases investment cost
  - almost infeasible with submarine cables.

Sources

[1] C. Bayliss and B. Hardy, “Transmission and distribution Electrical Engineering”, Elsevier, 2012, 4th edition, ISBN 978-0-08-096912-1

[2] P. Schavemaker, L. van der Sluis, “Electrical power system essentials”, John Wiley & Sons, 2008, ISBN 978-0470-51027-8