LIÈGE université
Sciences Appliquées

ELEC0014 - Introduction to electric power and energy systems

The synchronous machine (simplified model) – Part 2 –

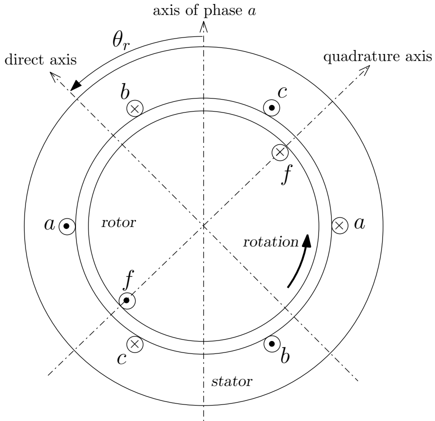
Thierry Van Cutsem
t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

November 2019

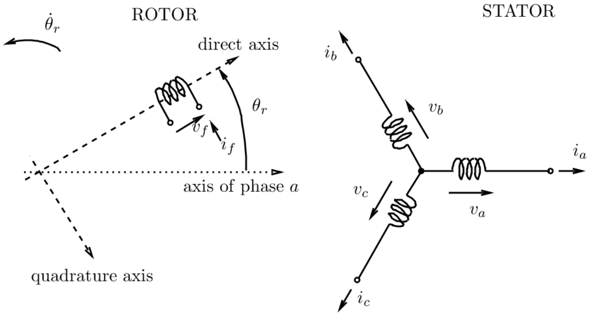
Modelling of machine with magnetically coupled circuits

Simplifying assumptions

- round rotor
- saturation of magnetic material neglected
- on the rotor: field winding only (acceptable since focus is on steady-state operation)
- single pair of poles (does not affect the electrical behaviour)



Relations between voltages, fluxes and currents



Stator :

$$v_a = -R_a i_a - \frac{d\psi_a}{dt} \qquad v_b = -R_a i_b - \frac{d\psi_b}{dt} \qquad v_c = -R_a i_c - \frac{d\psi_c}{dt}$$

R_a : resistance of each phase $\qquad$ ψ_a, ψ_b and ψ_c : flux linkages in phases

Field winding :

$$v_f = R_f i_f + \frac{d\psi_f}{dt}$$

R_f : resistance of winding $\qquad$ ψ_f : flux linkage in winding

Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \underbrace{\begin{bmatrix} L_o & -L_m & -L_m & L_{af}\cos\theta_r \\ -L_m & L_o & -L_m & L_{af}\cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af}\cos(\theta_r - \frac{4\pi}{3}) \\ L_{af}\cos\theta_r & L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{af}\cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix}}_{\mathbf{L}(\theta_r)} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

where $L_o, L_m > 0$.

- Self-inductance of any stator winding is constant (due to round rotor)
- mutual inductance between any two phases is constant (due to round rotor)
- ...and negative since a positive current i_x in phase x creates a negative flux ψ_y in phase y $(x \neq y)$
- self-inductance of field winding is constant (path of magnetic field identical whatever the position of the rotor)
- mutual inductance between one phase and the field winding is maximum and positive when $\theta_r = 0$, zero when $\theta_r = \frac{\pi}{2}$, minimum and negative when $\theta_r = \pi$.

Machine in steady-state operation

- rotation speed equal to nominal angular frequency :

$$\dot{\theta}_r = \omega_N \qquad \theta_r = \theta_r^o + \omega_N t$$

θ_r^o : rotor positon at $t = 0$

- constant direct current in field winding : $i_f = I_f$
- balanced three-phase voltages and currents in stator :

$$v_a(t) = \sqrt{2}V\cos(\omega_N t + \theta) \qquad i_a(t) = \sqrt{2}I\cos(\omega_N t + \psi)$$
$$v_b(t) = \sqrt{2}V\cos(\omega_N t + \theta - \tfrac{2\pi}{3}) \qquad i_b(t) = \sqrt{2}I\cos(\omega_N t + \psi - \tfrac{2\pi}{3})$$
$$v_c(t) = \sqrt{2}V\cos(\omega_N t + \theta - \tfrac{4\pi}{3}) \qquad i_c(t) = \sqrt{2}I\cos(\omega_N t + \psi - \tfrac{4\pi}{3})$$

with the corresponding phasors :

$$\bar{V} = V\, e^{j\theta}$$
$$\bar{I} = I\, e^{j\psi}$$

Flux linkage in one stator winding (phase a)

$$\psi_a = L_o\sqrt{2}I\cos(\omega_N t+\psi) - L_m\sqrt{2}I\cos(\omega_N t+\psi-\frac{2\pi}{3}) - L_m\sqrt{2}I\cos(\omega_N t+\psi-\frac{4\pi}{3}) + L_{af}\cos(\omega_N t+\theta_r^o)\, I_f$$

Adding and subtracting $L_m\sqrt{2}I\cos(\omega_N t+\psi)$ yields :

$$\begin{aligned}\psi_a &= L_o\sqrt{2}I\cos(\omega_N t+\psi) + L_m\sqrt{2}I\cos(\omega_N t+\psi) \\ &\quad - L_m\sqrt{2}I\underbrace{\left(\cos(\omega_N t+\psi)+\cos(\omega_N t+\psi-\frac{2\pi}{3})+\cos(\omega_N t+\psi-\frac{4\pi}{3})\right)}_{=0} \\ &\quad + L_{af}I_f\cos(\omega_N t+\theta_r^o) \\ &= \underbrace{\sqrt{2}(L_o+L_m)I\cos(\omega_N t+\psi)}_{\psi_a^s} + \underbrace{L_{af}I_f\cos(\omega_N t+\theta_r^o)}_{\psi_a^r} \qquad (1)\end{aligned}$$

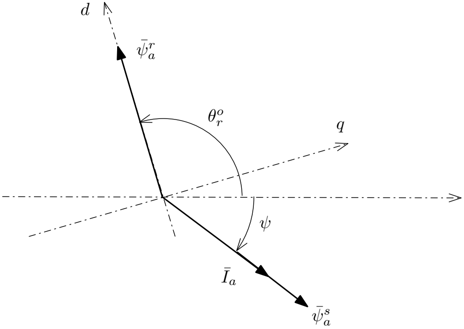
ψ_a^s : flux of the rotating field produced by the three stator currents

ψ_a^r : flux of the field created by the current i_f

Both flux components being sinusoidal functions of time (with angular frequency ω_N), they can be characterized by phasors :

$$\bar{\psi}_a^s = (L_o + L_m)\ I\ e^{j\psi} \qquad \bar{\psi}_a^r = \frac{L_{af}}{\sqrt{2}}\ I_f\ e^{j\theta_r^o}$$

Phasor diagram :



Horizontal axis

- = axis on which rotating vectors are projected
- = axis to which the rotor position is referred, i.e. axis of phase a

Flux linkage in field winding

$$
\begin{aligned}
\psi_f = L_{ff} I_f \quad &+ \quad L_{af} \cos(\omega_N t + \theta_r^o)\, \sqrt{2} I \cos(\omega_N t + \psi) \\
&+ \quad L_{af} \cos(\omega_N t + \theta_r^o - \frac{2\pi}{3})\, \sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3}) \\
&+ \quad L_{af} \cos(\omega_N t + \theta_r^o - \frac{4\pi}{3})\, \sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3}) \\
= L_{ff} I_f \quad &+ \quad \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)\right] \\
&+ \quad \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi - \frac{4\pi}{3})\right] \\
&+ \quad \frac{\sqrt{2} L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi + \frac{4\pi}{3})\right] \\
= \underbrace{L_{ff} I_f}_{\psi_f^r} \quad &+ \quad \underbrace{\frac{3\sqrt{2} L_{af}}{2} I \cos(\theta_r^o - \psi)}_{\psi_f^s}
\end{aligned}
$$

ψ_f^s : flux of the rotating field produced by the three stator currents; constant magnitude; at an angle $\theta_r^o - \psi$ wrt to field winding

ψ_f^r : flux created by field current

Voltage-current relation at stator

Replacing v_a, i_a and ψ_a by their expressions :

$$\sqrt{2}V\cos(\omega_N t+\theta) = -R_a\sqrt{2}I\cos(\omega_N t+\psi)+\sqrt{2}\omega_N(L_o+L_m)I\sin(\omega_N t+\psi) + \sqrt{2}\frac{\omega_N L_{af}}{\sqrt{2}} I_f \sin(\omega_N t+\theta_r^o)$$

Let us define :

$X=\omega_N(L_o+L_m)$: the *synchronous reactance* of the machine[1]

$E_q=\frac{\omega_N L_{af}}{\sqrt{2}} I_f$: RMS value of an e.m.f. proportional to field current I_f

The above equation becomes:

$$\sqrt{2}V\cos(\omega_N t+\theta) = -R_a\sqrt{2}I\cos(\omega_N t+\psi)+\sqrt{2}XI\cos(\omega_N t+\psi-\frac{\pi}{2}) +\sqrt{2}E_q\cos(\omega_N t+\theta_r^o-\frac{\pi}{2})$$

[1] $R_a + jX$ is the *cyclic impedance* defined in a previous lecture

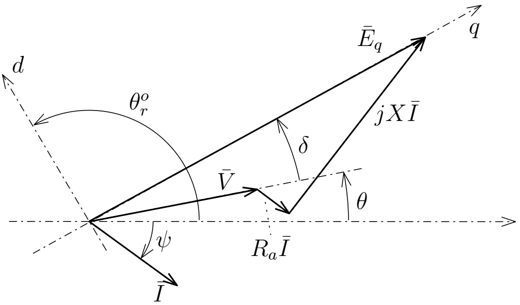
The corresponding phasor equation is:

$$V e^{j\theta} = -R_a I \, e^{j\psi} + XI \, e^{j\psi} e^{-j\frac{\pi}{2}} + E_q \, e^{j(\theta_r^o - \frac{\pi}{2})}$$

or simply:

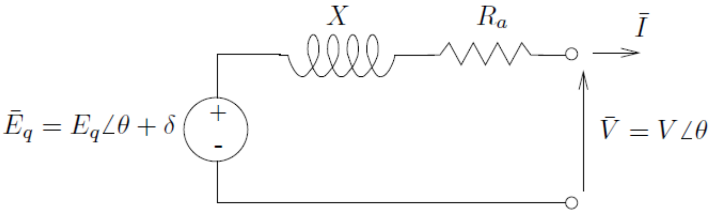
$$\bar{V} = -R_a \, \bar{I} - jX \, \bar{I} + \bar{E}_q$$

where $\bar{E}_q = E_q e^{j(\theta_r^o - \frac{\pi}{2})}$ is the phasor of the e.m.f. E_q, lying on the q axis[2]



[2]hence the notation E_q

Per-phase equivalent circuit



The synchronous reactance X characterizes the steady-state operation of the machine.

δ is the phase shift between the internal e.m.f. $\bar{E}_q$ and the terminal voltage $\bar{V}$.

δ is called the *internal angle* or *load angle* of the machine.

Nominal values and orders of magnitude

- Nominal voltage U_N: voltage for which the machine has been designed (in particular its insulation).
  The real voltage may deviate from this value by a few %
- nominal current I_N: current for which machine has been designed (in particular the cross-section of its conductors).
  Maximum current that can be accepted without limit in time
- nominal apparent power:

$$S_N = \sqrt{3} U_N I_N$$

Machine parameters in per unit on the base $(S_B = S_N, V_B = U_N/\sqrt{3})$:

$R_a \simeq 0.005$ pu

$X \simeq 1.5 - 2.5$ pu (for a round-rotor machine as considered in this lecture)

Powers in synchronous machine : general case

Power balance of the stator

$$p_{r \to s} = p_T + p_{Js} + \frac{dW_{ms}}{dt}$$

$p_{r \to s}$: power transfer from rotor to stator

p_T : three-phase instantaneous power leaving the stator

p_{Js} : Joule losses in stator windings

W_{ms} : magnetic energy stored in the stator windings

Nature of $p_{r \to s}$?

- mechanical power for sure (torque applied to rotating masses)
- is there some electromagnetic transfer of power (like in a transformer) ?

Power balance of the rotor

$$p_f + P_m = p_{Jf} + \frac{dW_{mf}}{dt} + \frac{dW_c}{dt} + p_{r \to s}$$

P_m : mechanical power provided by the turbine
p_f : electrical power provided to the field winding by the excitation system
p_{Jf} : Joule losses in the field winding
W_{mf} : magnetic energy stored in the field winding
W_c : kinetic energy of all rotating masses (generator and turbine)

Total electromagnetic energy stored in the machine :

$$\begin{aligned} W_{m\,tot} &= \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \mathbf{L}(\theta_r) \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} = \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} \\ &= \underbrace{\frac{1}{2} (i_a \psi_a + i_b \psi_b + i_c \psi_c)}_{W_{ms}} + \underbrace{\frac{1}{2}\, i_f \psi_f}_{W_{mf}} \end{aligned}$$

Motion equation

$$\mathcal{I}\,\frac{d^2\theta_r}{dt^2} = T_m - T_e$$

- $\mathcal{I}$: moment of inertia of all rotating masses
- T_m : mechanical torque applied to the rotor by the turbine
- T_e : electromagnetic torque applied to the rotor by the generator.

Multiplying by the rotor speed $d\theta_r/dt$:

$$\mathcal{I}\,\frac{d\theta_r}{dt}\,\frac{d^2\theta_r}{dt^2} = \frac{d\theta_r}{dt}\,T_m - \frac{d\theta_r}{dt}\,T_e$$

$$\Leftrightarrow \qquad \frac{dW_c}{dt} = P_m - \frac{d\theta_r}{dt}\,T_e$$

and the power balance of the rotor becomes :

$$p_f + \frac{d\theta_r}{dt}\,T_e = p_{Jf} + \frac{dW_{mf}}{dt} + p_{r\rightarrow s}$$

Powers in synchronous machine : steady-state operation

Power balance of stator

$$\begin{aligned}\frac{1}{2} i_a \psi_a &= (L_o + L_m) I^2 \cos^2(\omega_N t + \psi) + \frac{\sqrt{2}}{2} L_{af} I_f I \cos(\omega_N t + \theta_r^o) \cos(\omega_N t + \psi) \\ &= \frac{1}{2}(L_o + L_m) I^2 + \frac{1}{2}(L_o + L_m) I^2 \cos(2\omega_N t + 2\psi) + \\ &\quad \frac{\sqrt{2}}{4} L_{af} I_f I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi) \right]\end{aligned}$$

By doing the same derivation for phases b and c, and adding all three results :

$$W_{ms} = \frac{1}{2}\left(i_a \psi_a + i_b \psi_b + i_c \psi_c\right) = \frac{3}{2}(L_o + L_m) I^2 + \frac{3\sqrt{2}}{4} L_{af} I_f I \cos(\theta_r^o - \psi)$$

$$W_{ms} \text{ is constant, i.e. } \frac{dW_{ms}}{dt} = 0.$$

In three-phase balanced operation :

$$p_T = 3P$$

where P is the active power produced by one phase.

Hence, the power balance of the stator simply becomes :

$$p_{r \to s} = 3\,P + p_{Js}$$

Power balance of rotor

$$W_{mf} = \frac{1}{2} i_f \psi_f = \frac{1}{2} L_{ff} I_f^2 + \frac{3\sqrt{2}}{4} L_{af} I \, I_f \cos(\theta_r^o - \psi)$$

$$W_{mf} \text{ is constant, i.e. } \frac{dW_{mf}}{dt} = 0.$$

$$\frac{d\psi_f}{dt} = 0 \quad \Rightarrow \quad V_f = R_f I_f \quad \Rightarrow \quad p_f = R_f \, I_f^2 = p_{Jf}$$

In steady state, the power entering the field winding is dissipated in Joule losses !

The field current aims at "magnetizing" the rotor, allowing the torque T_e to be created, but the field winding does not exchange power with the other windings.

$$\frac{d\theta_r}{dt} = \omega_N \qquad \frac{dW_c}{dt} = 0 \qquad T_m = T_e \qquad P_m = \omega_N T_e = \omega_N T_m$$

Hence, the power balance of the rotor simply becomes :

$$p_{r\rightarrow s} = \omega_N T_e = \omega_N T_m = P_m$$

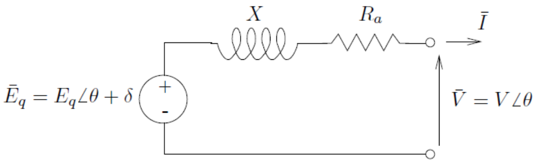
The power $p_{r\rightarrow s}$ transfered from rotor to stator is purely mechanical !

Expression of active and reactive powers

We assume $R_a \simeq 0$.

Active and reactive powers produced by the machine, in per unit :

$$
\begin{aligned}
P &= -\frac{VE_q}{X}\sin(\theta - (\theta+\delta)) = \frac{VE_q}{X}\sin\delta \\
Q &= -\frac{V^2}{X} + \frac{VE_q}{X}\cos(\theta - (\theta+\delta)) = -\frac{V^2}{X} + \frac{VE_q}{X}\cos\delta
\end{aligned}
$$

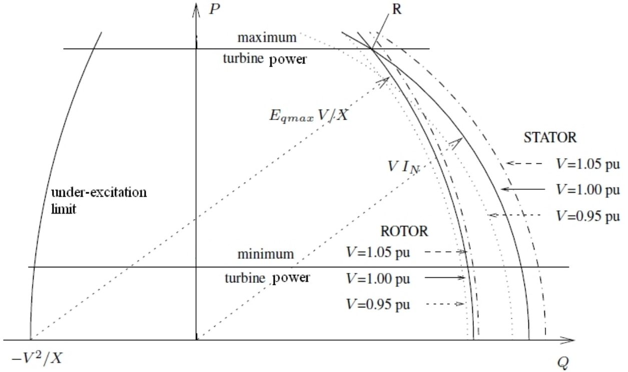


Capability curves

Seen from the network, a generator is characterized by three variables: V, P et Q

Limits on P, Q et V corresponding to generator operation within limits ?

The *capability curves* define the set of admissible operating points in the (P, Q) space, under constant voltage V (justified by automatic voltage regulator)



Stator (heating) limit

stator current $I = I_N$ in per unit: $(S^2 =)\ P^2 + Q^2 = V^2\, I_N^2$

Rotor (heating) limit

field current $I_f = I_{fmax}$ $\Rightarrow$ $E_q = E_{qmax} = \dfrac{\omega_N L_{af}}{\sqrt{2}} I_{fmax}$

With the same simplifying assumptions as before, and with $R_a = 0$:

$$P = \frac{E_{qmax} V}{X} \sin\delta \qquad Q = \frac{E_{qmax} V}{X} \cos\delta - \frac{V^2}{X}$$

after eliminating δ:

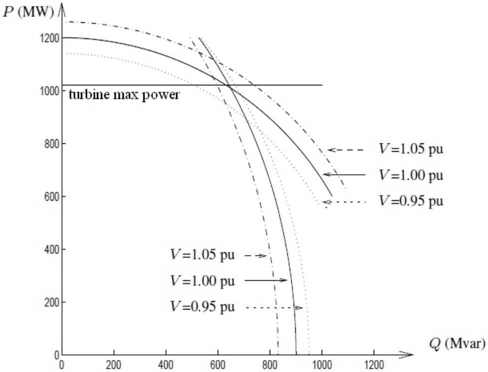
$$\left(\frac{VE_{qmax}}{X}\right)^2 = \left(Q + \frac{V^2}{X}\right)^2 + P^2$$

- Lower limit on active power caused by stability of combustion in thermal power plants
- maximum reactive power increases when the active power decreases
  - to relieve an overloaded machine, P can be decreased but this power has to be produced by some other generators !
- for a given value of P, the maximum reactive power increases with V
  - this holds true under the simplifying assumption of a non saturated machine; see next slide for a case with saturation
- in practice, under $V = 1$ pu, the two-by-two intersection points of respectively the turbine, the rotor and the stator limits are close to each other ("coherent" design of stator and rotor)
- the stator limits can be increased by a stronger cooling (e.g. higher hydrogen pressure in stator windings).

Under-excitation limit

Corresponds to a stability, not a thermal limit: absorbing more $Q \Rightarrow$ decreasing E_q $\Rightarrow$ decreasing $i_f \rightarrow$ maximum torque T_e decreases $\Rightarrow$ risk of losing synchronism. See Chapter on voltage regulation.

Capability curves ($Q > 0$ part only) of a real machine with saturation taken into account



- the overall shape of the curves is the same
- but the rotor limit becomes more constraining when V increases.