ELEC0014 - Introduction to power and energy systems

Balanced three-phase systems and operation

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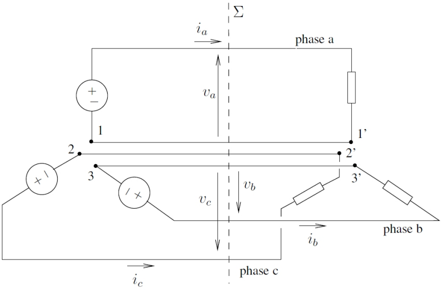
- system used for the major part of transmission and distribution of electric energy (except for high-voltage direct current links)
- advantages: saving of conductors, production of rotating magnetic fields in machines

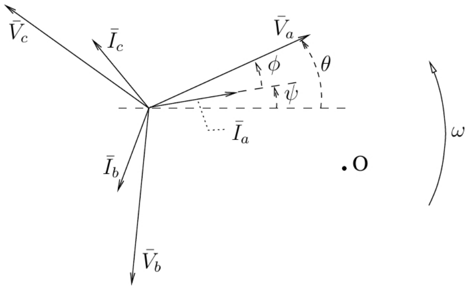
- balanced three-phase circuit = an assembly of three identical circuits called *phases*
- balanced three-phase operation: at the corresponding points of the three phases, the voltages (resp. the currents) have the same effective (or RMS) value but are shifted in time by one third of a period from one phase to another.

Principle



$$
\begin{aligned}
v_a(t) &= \sqrt{2}V\cos(\omega t+\theta) \\
v_b(t) &= \sqrt{2}V\cos(\omega(t-\frac{T}{3})+\theta) = \sqrt{2}V\cos(\omega t+\theta-\frac{2\pi}{3}) \\
v_c(t) &= \sqrt{2}V\cos(\omega(t-\frac{2T}{3})+\theta) = \sqrt{2}V\cos(\omega t+\theta-\frac{4\pi}{3}) \\
i_a(t) &= \sqrt{2}I\cos(\omega t+\psi) \\
i_b(t) &= \sqrt{2}I\cos(\omega(t-\frac{T}{3})+\psi) = \sqrt{2}I\cos(\omega t+\psi-\frac{2\pi}{3}) \\
i_c(t) &= \sqrt{2}I\cos(\omega(t-\frac{2T}{3})+\psi) = \sqrt{2}I\cos(\omega t+\psi-\frac{4\pi}{3})
\end{aligned}
$$

Phasor diagrams



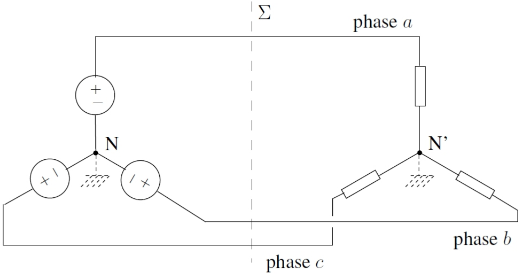
$$\begin{aligned}
\bar{V}_a &= Ve^{j\theta} \\
\bar{V}_b &= Ve^{j(\theta-\frac{2\pi}{3})} = \bar{V}_a e^{-j\frac{2\pi}{3}} \\
\bar{V}_c &= Ve^{j(\theta-\frac{4\pi}{3})} = \bar{V}_a e^{-j\frac{4\pi}{3}} = \bar{V}_b e^{-j\frac{2\pi}{3}} \qquad \Longrightarrow \qquad \bar{V}_a + \bar{V}_b + \bar{V}_c = 0 \\
\bar{I}_a &= Ie^{j\psi} \\
\bar{I}_b &= Ie^{j(\psi-\frac{2\pi}{3})} = \bar{I}_a e^{-j\frac{2\pi}{3}} \\
\bar{I}_c &= Ie^{j(\psi-\frac{4\pi}{3})} = \bar{I}_a e^{-j\frac{4\pi}{3}} = \bar{I}_b e^{-j\frac{2\pi}{3}} \qquad \Longrightarrow \qquad \bar{I}_a + \bar{I}_b + \bar{I}_c = 0
\end{aligned}$$

An observer located in O sees the rotating vectors passing in the order *a*, *b*, *c*

$\bar{V}_a$, $\bar{V}_b$, $\bar{V}_c$ make up a *direct sequence*

A more interesting configuration

The return conductors 11', 22' et 33' are merged into a single conductor. . . which carries a current $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$ . . . and, hence, is removed



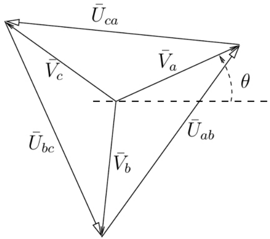
The power through Σ = 3 times the power in one phase, using only 1.5 times the number of conductors in one phase.

N, N' : *neutrals*. In balanced three-phase operation, all neutrals are at the same voltage.

$\bar{V}_a$, $\bar{V}_b$, $\bar{V}_c$: *phase* or *phase-to-neutral* or *line-to-neutral voltages*

Line voltages (or voltages between phases)

$$\bar{U}_{ab} = \bar{V}_a - \bar{V}_b \qquad \bar{U}_{bc} = \bar{V}_b - \bar{V}_c \qquad \bar{U}_{ca} = \bar{V}_c - \bar{V}_a$$



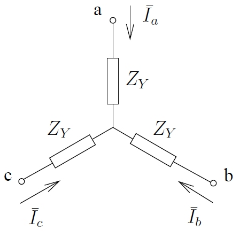
$$\bar{U}_{ab} = \sqrt{3}\ \bar{V}_a\ e^{j\frac{\pi}{6}} = \sqrt{3}\ V\ e^{j(\theta+\frac{\pi}{6})} \qquad \bar{U}_{bc} = \sqrt{3}\ \bar{V}_b\ e^{j\frac{\pi}{6}} \qquad \bar{U}_{ca} = \sqrt{3}\ \bar{V}_c\ e^{j\frac{\pi}{6}}$$

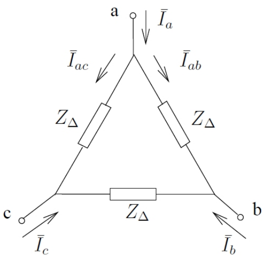
RMS value of line voltage $= \sqrt{3} \times$ RMS value of phase voltage

$\bar{U}_{ab}$, $\bar{U}_{bc}$ et $\bar{U}_{ca}$ also make up a direct sequence.

When specifying the voltage at the terminal of a three-phase device, unless otherwise specified, it is the effective (or RMS) value of the line voltages.

Star and delta (or triangle) connections





Relation between $\bar{I}_{ab}$ and $\bar{I}_a$ in the delta connection ?

$$\bar{I}_a = \frac{\bar{U}_{ab} + \bar{U}_{ac}}{Z_\Delta} = \frac{\bar{U}_{ab} - \bar{U}_{ca}}{Z_\Delta} = \frac{\bar{U}_{ab} - \bar{U}_{ab} e^{-j\frac{4\pi}{3}}}{Z_\Delta} = \frac{\bar{U}_{ab}}{Z_\Delta}(1 - e^{-j\frac{4\pi}{3}}) = \sqrt{3}\, e^{-j\frac{\pi}{6}}\, \bar{I}_{ab}$$

If the same voltages $\bar{V}_a$, $\bar{V}_b$ and $\bar{V}_c$ are applied to both schemes, the phase currents $\bar{I}_a$, $\bar{I}_b$ et $\bar{I}_c$ will be also the same if : $Z_\Delta = 3\, Z_Y$

Exercise. Establish this result by expressing that both circuits consume the same three-phase complex power.

In practice:

1. single phase loads are placed in the branches of either stars or deltas, according to the desired voltage
   - example: three-phase power supply at 400 V (between phases): appliances designed to operate under 230 V are placed between one phase and the neutral
2. at the level of one house:
   - modern distribution: three-phase supply
   - the distribution company provides a neutral conductor
   - the various single-phase loads are connected to the phases so that the latter are balanced as much as possible
   - there remains some phase imbalance $\longrightarrow$ nonzero current in the neutral conductor
3. as more and more houses are supplied, the total neutral current becomes negligible compared to the phase currents
4. most of the inverters used in residential photovoltaic units are single-phase units; they give rise to imbalances
5. however, most of the loads seen from the transmission network can be considered to be balanced (imbalance cancellation due to large number of individual loads involved).

Per-phase analysis

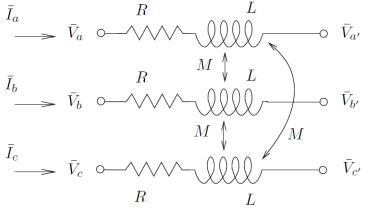
Balanced three-phase operation $\Rightarrow$ consider only one of the three phases.

Voltages and current phasors in the other phases are merely shifted by $\pm 2\pi/3$ rad.

In order to handle a single phase and discard the other two :

- the delta-connected three-phase loads must be replaced by their star-connected equivalents
- the inductive and capacitive couplings between phases must be taken into account.

Handling of inductive couplings between phases



Approximation: assume perfectly balanced couplings:

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} \bar{V}_{a'} \\ \bar{V}_{b'} \\ \bar{V}_{c'} \end{bmatrix} + \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

Developing the first component:

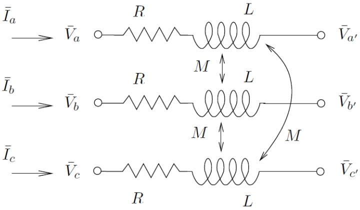
$$\begin{aligned} \bar{V}_a &= \bar{V}_{a'} + Z_s \bar{I}_a + Z_m \bar{I}_b + Z_m \bar{I}_c \\ &= \bar{V}_{a'} + \left[Z_s + Z_m (e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}) \right] \bar{I}_a \\ &= \bar{V}_{a'} + [Z_s - Z_m] \, \bar{I}_a \end{aligned}$$

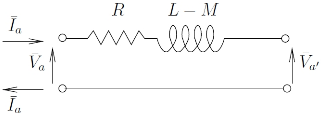
Thus, phase *a* can be considered alone but with a series impedance:

$$Z_{eq} = Z_s - Z_m$$

named *cyclic impedance*.

Example of previous slide:

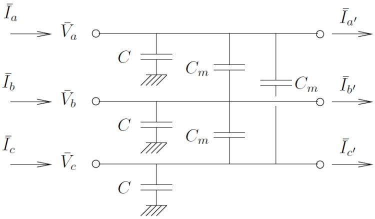




$$Z_{eq} = R + j\omega(L - M)$$

Remark. The return conductor is needed to carry the return current in the equivalent circuit; it does not exist in the original three-phase circuit!

Handling of capacitive couplings between phases



Approximation: assume perfectly balanced couplings:

$$\begin{bmatrix} \bar{I}_a - \bar{I}_{a'} \\ \bar{I}_b - \bar{I}_{b'} \\ \bar{I}_c - \bar{I}_{c'} \end{bmatrix} = \begin{bmatrix} Y_s & Y_m & Y_m \\ Y_m & Y_s & Y_m \\ Y_m & Y_m & Y_s \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

Developing the first component:

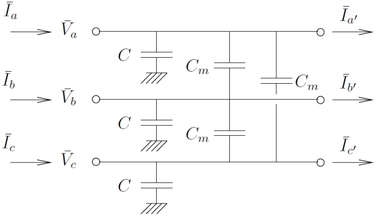
$$\begin{aligned} \bar{I}_a - \bar{I}_{a'} &= Y_s \bar{V}_a + Y_m \bar{V}_b + Y_m \bar{V}_c \\ &= \left[Y_s + Y_m (e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}) \right] \bar{V}_a \\ &= [Y_s - Y_m] \, \bar{V}_a \end{aligned}$$

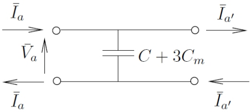
Thus, phase *a* can be considered alone but with a shunt admittance:

$$Y_{eq} = Y_s - Y_m$$

between phase and neutral.

Example of previous slide:

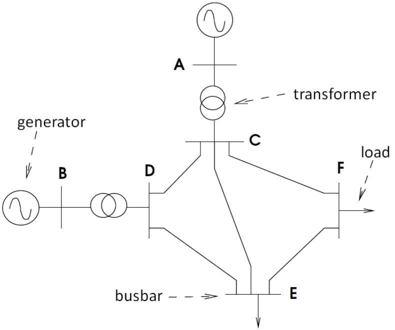




$$\begin{aligned} Y_s &= j\omega C + 2j\omega C_m \\ Y_m &= -j\omega C_m \\ \text{and hence:} \quad Y_{eq} &= j\omega(C + 3C_m) \end{aligned}$$

One-line diagram

Single-phase diagram without return conductor



In the switching stations, lines, cables, transformers, generators, loads, etc. are connected to each other through *buses* or *busbars* [1]

Bus (or busbar) = equipotential metallic assembly

[1] En français, l'ensemble des barres relatives aux trois phases est appelé *jeu de barres*

Powers in balanced three-phase systems

Instantaneous power through the section Σ:

$$\begin{aligned}
p(t) &= v_a i_a + v_b i_b + v_c i_c \\
&= 2VI \Big[\cos(\omega t + \theta)\cos(\omega t + \psi) + \cos(\omega t + \theta - \frac{2\pi}{3})\cos(\omega t + \psi - \frac{2\pi}{3}) \\
&\qquad + \cos(\omega t + \theta - \frac{4\pi}{3})\cos(\omega t + \psi - \frac{4\pi}{3})\Big] \\
&= 3VI\cos(\theta - \psi) + VI\Big[\cos(2\omega t + \theta + \psi) \\
&\qquad + \cos(2\omega t + \theta + \psi - \frac{4\pi}{3}) + \cos(2\omega t + \theta + \psi - \frac{2\pi}{3})\Big] \\
&= 3VI\cos(\theta - \psi) = 3VI\cos\phi = 3P
\end{aligned}$$

- no fluctuating power!
- instantaneous power $= 3 \times$ active power transfer P in one of the phases
- in each phase there is a fluctuating power. The magnitude of its component relative to reactive current is the single-phase reactive power Q.

Complex three-phase power

By extension of the single-phase formula:

$$\begin{aligned} S_{tri} &= \bar{V}_a \bar{I}_a^\star + \bar{V}_b \bar{I}_b^\star + \bar{V}_c \bar{I}_c^\star \\ &= \bar{V}_a \bar{I}_a^\star + \bar{V}_a e^{-j\frac{2\pi}{3}} \bar{I}_a^\star e^{j\frac{2\pi}{3}} + \bar{V}_a e^{-j\frac{4\pi}{3}} \bar{I}_a^\star e^{j\frac{4\pi}{3}} \\ &= 3\bar{V}_a \bar{I}_a^\star \end{aligned}$$

Three-phase active power : $re\,(S_{tri}) = P_{tri} = 3VI\cos\phi = 3P$

Three-phase reactive power: $im\,(S_{tri}) = Q_{tri} = 3VI\sin\phi = 3Q$

Q_{tri} is an artificial notion :

- since there is no three-phase fluctuating power
- only the single-phase reactive power Q has an interpretation
- it is as artificial as would be a "three-phase current" $3I$

However, the three-phase reactive power is used worldwide, for its symmetry with the three-phase active power.

Expressions involving the line voltage

To designate the voltage in a three-phase system, one uses generally:

$$U = \text{effective (or RMS) value of line voltage} = \sqrt{3}V$$

$$\begin{aligned} P_{tri} &= \sqrt{3}UI\cos\phi \\ Q_{tri} &= \sqrt{3}UI\sin\phi \end{aligned}$$

These formulae are "hybrid" in so far as:

- U is the effective value of the *line voltage*
- while ϕ is the phase angle between the line current and the *phase-to-neutral* voltage.